SYLLABUS FOR SPRING 2019 MAT 569: ELLIPTIC OPERATORS AND TOPOLOGY

FRANCESCO LIN

The protagonist of this class is the interaction between topology, geometry and spectral theory. Our long term goal is to understand the Atiyah-Patodi-Singer index theorem for Dirac operators on manifolds with boundary, but we will take a panoramic route and discuss many incarnations of this interaction.

Schedule. Monday and Wednesday, 9:30-10:50AM.

Contents. This class will cover the following topics:

• Week 1: Review of elliptic operators on compact manifolds
• Week 2-3: geometry of the Laplacian on functions - hyperbolic surfaces
• Week 4: Characteristic classes from the point of view of differential forms
• Week 5-6: Spin geometry
• Week 7-8: index theorem and its applications
• Week 9-10: heat kernels and traces
• Week 11: the case of manifolds with boundary
• Week 12: spectra of compact Riemann surfaces

Grading. There will be biweekly homework assignments. The students will also work on a final project, probably a 5-10 page expository paper.

References. We will mostly follow these books:

• Buser - Geometry and spectra of compact Riemann surfaces
• Chavel - Eigenvalues in Riemannian geometry
• Martelli - An introduction to Geometric topology
• Roe - Elliptic operators, topology, and asymptotic methods
• Shanahan - The Atiyah-Singer index theorem
• Warner - Foundations of Differentiable Manifolds and Lie Groups

Along the way, we will also look at several papers, including:

• Atiyah, Patodi, Singer - Spectral asymmetry and Riemannian geometry I
• Gromov, Lawson - Spin and scalar curvature in the presence of fundamental group I
• Hitchin - Harmonic spinors
• Long, Reid - On the geometric boundaries of hyperbolic four-manifolds

Prerequisites. As the main topic of the class is the interaction between topology, geometry and spectral theory, I will assume that people know the following: characteristic classes, connections on bundles and their curvature, the spectral theorem for self-adjoint compact operators. Feel free to contact me in case you need references for these topics.