Calculus I
Section 005
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Midterm 1: What you need to know, and some tips

Chapter 1: It is expected that you are familiar with the concept of a function, descriptions of functions, and graphs of functions. Be familiar with the basic functions (polynomials, rationals, roots, exponentials, logs, trig and inverse trigs), and know how to evaluate them in simple cases. In particular, know sin, cos and tan of 0, $\pi/6$, $\pi/4$, $\pi/3$ and $\pi/2$. Also, know how to use the "equivalent equation" to evaluate inverse trig functions, but don't worry about memorizing the ranges of these functions (i.e. if you need to know what quadrant $\sin^{-1} - \frac{1}{2}$ is in, you will be reminded).

Understand how to compose functions. Have a good idea of how to find the domain of a composition of two functions. In general this can be a difficult task, but if you think of functions as machines, the idea is to see what numbers you can pass through the first machine and into the second, and get an output without breaking either machine (by doing something like inputting $x = 0$ to $f(x) = \frac{1}{x}$).

Also, understand inverse functions, what they mean, when they exist, how we might go about finding them, and equivalent equations. Also, wouldn't hurt to be familiar with how to draw the graph of the inverse function given the graph of the function itself.

Section 2.1: See section 2.7/2.8; 2.1 is mostly an easier version of 2.7 that is placed in the beginning simply to motivate the study of limits.

Section 2.2: Understand the notion of limit intuitively, numerically, and graphically as best you can. If you feel comfortable with those, then try to grapple with what the book's definition of limit means. Be aware that while we try to understand limits by seeing what number the values of $f(x)$ "lead up to", the value of a limit is a precise number.

Remember, the purpose of limits is to give a good way to talk about what the value of a function "should be"; e.g. if $f(x) = \frac{x}{x}$, $f(0)$ "should be" 1, but instead it is undefined, so we cope by saying $\lim_{x\to 0} f(x) = 1$. (In fact, you can think of continuity as when what the function "should be" agrees with what it actually is, but we'll talk about continuity later.)

Know about left and right hand limits, and infinite limits. Know that even though we may write that a certain limit "equals infinity", that the limit does not actually exist. Nonetheless, be able to determine when there are infinite limits. Know about vertical asymptotes.

Know that the limit exists if and only if the left and right hand limits exist and are equal. This will be key to figuring out whether or not limits exist for functions described by their graphs or piecewise-defined functions.

Section 2.3: So the first thing about this section is the 239,107 limit laws confronting you. Definitely know laws 7 and 8 (not by those names, know the facts contained in them). As for the others, it might be advisable to think of them not as separate laws, so much as instances of the general principle: if new function = doing something to old functions, then new limit = doing same thing to old limits, as long as old limits exist. The one major exception to that rule is composing functions, so beware of that – you might need to think about continuity.

Know about the direct substitution property – which is actually much more general than that stated on page 102, as you can replace "polynomial or rational function" with any sum, product, or composition of polynomials, rationals, roots, exponentials, logs, trig and inverse trigs. This enables you to evaluate lots of limits very easily.

Be aware of what I called the "wannabe" property, and the book so colorfully calls "red box near top of page 103". If you try to use the direct substitution property but end up with something like 0/0, then you're gonna need a trick, and this fact. The kind of tricks you need include such hits as factoring the top and bottom of a fraction and cancelling; multiplying top and bottom of a fraction times a conjugate; and adding together two or more fractions by finding a common denominator. A lot of stuff with fractions, I guess.

For the aforementioned tricks, remember: we are never substantially changing the expression we are looking at, just changing what it looks like by multiplying by an expression that is equal to 1 at all but maybe a couple of numbers. So don't try to, say, square the top and the bottom of an expression containing square roots: that squares the expression, which does change it.

Also, knowing the squeeze theorem would be nice.

Section 2.4: Didn't do this one.

Section 2.5: Intuitively, know what continuity at a point or on an interval means ("don't have to pick up pencil" or "what function should be = what function is"), and know how it is defined.

Know theorem 7 on page 124, about how all the usual suspects are continuous on their domains. Also, know theorems 8 and 9 on page 125. For theorem 8, there might be a question on the exam about how one thing involving a limit doesn't equal another thing involving a limit, even though they look similar, and I might ask you how this can be, and the answer might be that this or that function isn't continuous at a point. Just saying.

Also, know the intermediate value theorem, and try to have some idea about using it to estimate solutions of equations (i.e. showing there is a solution between two given x values).

Section 2.6: Know limits at infinity, the difference between them and infinite limits, and in particular that these guys generally DO exist (you know, as long as there is a finite value being approached). Know about horizontal asymptotes.

As for evaluating these guys, you might have noticed that I was kind of ambivalent about where the draw line between "not rigorous enough" and "simply pedantic". So, let me clarify: use the tricks from class (multiply by $\frac{\text{conjugate}}{\text{conjugate}}$ or $\frac{1}{x^n}/\frac{1}{x^n}$, and $\lim_{x\to\pm\infty}\frac{1}{x^n}=0$) to simplify. Do this until you are left with a fraction where you can find the limits of the top and bottom, and they aren't both zero or both infinity. From there, I'm fine with you just extracting the answer: if the ratio of top to bottom is a number, swell; if the bottom goes to infinity, the whole fraction goes to zero; if the bottom goes to zero, there will be an infinite limit. If this doesn't make sense, ask me, and I'll clarify it.

Section 2.7: Ideally, you would understand how both the questions we raise and answers we give are very natural and organic and whatever hippy-dippy words you want to use to describe them. (And don't get me wrong, that would make me very happy as a teacher).

Realistically, you just need to understand the questions and the answers. The questions being:

a)how can I measure a rate of change over an instant? and
b)how can I draw a line that *best* approximates a curve?;

and the answers being:

a)take the derivative, and
b)take the derivative.

More specifically:

a)the instantaneous rate of change of $f(x)$ at $x = a$ is the derivative of f at a, $f'(a)$, and
b)$f'(a)$ is the slope of the tangent to curve $y = f(x)$ at $(a, f(a))$.

Obviously, know how to compute derivatives, i.e. using the definition.

It would be helpful know how to find the units for the derivative in a given word problem, and also to have an idea how to interpret what it is the derivative measures.

Section 2.8: Understand that the derivative of a function can also be considered as a function, and know how to use the second definition of the limit to find this function. For this, remember that (using the definition with x and h, where h goes to 0) we treat x like a *constant*, and never try to, say, substitute anything for x while taking the limit; only h changes and approaches 0.

Be aware of how the graph of a function can be used to give a rough sketch of the graph of the derivative. Know the various notations for derivatives. Know about the notion of differentiability, and how it relates with continuity. Be familiar with the various ways a function can fail to be differentiable.

Section 3.1: Know the sum, difference, and constant multiplication rules; and know how to evaluate the derivatives of polynomials using these and the power rule. Tadaa.