Calculus I
Section 005
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Practice Midterm II

1. Find the derivatives of the following functions. You do not have to simplify your answers.

a. $f(x) = 3x^2$

b. $g(x) = \tan x + \cos x + \ln(x^2 - 1)$

c. $q(t) = \frac{1-e^t}{\sqrt[3]{1+t}}$

d. $h(x) = \frac{(x+2)^{20}\sqrt[5]{x-1}}{(x-49)(x-26)}$ (Hint: this can be done directly, but there's an easier way involving logarithms.)

Answers: **a.** $6x$
b. $\sec^2(x) - \sin(x) + \frac{2x}{x^2-1}$
c. $\frac{(1+t)^{\frac{1}{3}}(-e^t)-(1-e^t)(\frac{1}{3}(1+t)^{-\frac{2}{3}})}{(1+t)^{\frac{2}{3}}}$
d. $\frac{(x+2)^{20}\sqrt[5]{x-1}}{(x-49)(x-26)}(\frac{20}{x+2} + \frac{\frac{1}{5}}{x-1} - \frac{1}{x-49} - \frac{1}{x-26})$

2. Consider the equation $x^2y + xy^2 = 3x$.

a. Using implicit differentiation, compute $\frac{dy}{dx}$ in terms of x and y.

b. Find the equation of the tangent line to the curve given by this equation at the point (2,1).

Answers: a. $\frac{3-2xy-y^2}{x^2+2xy}$
b. $y - 1 = -\frac{1}{4}(x - 2)$

3. After an oil spill, a circular oil slick grows in such a way that its area increases at the rate of 15 $\frac{\text{mi}^2}{\text{hr}}$. When the area of the slick is 225π mi^2, at what rate is the radius of the slick increasing? (Remember that the area of a circle is given by $A = \pi r^2$.)

Answer: $\frac{1}{2\pi}\,\frac{\text{mi}}{\text{hr}}$

4. A bug crawls along a yard stick, starting at time $x = 0$ seconds. Its position at time x seconds is y inches, where $y = \sqrt{x+1} - \frac{1}{4}x + 8$.

a. In $\frac{\text{in}^2}{\text{sec}}$, what is the **acceleration** of the bug after 2 seconds?

b. At what time is the bug at rest?

c. Using a linear approximation (or differentials), estimate the bug's position at 0.08 seconds to 2 decimal places.

Answers: **a.** $-\frac{1}{4}(3^{-\frac{3}{2}}) = -\frac{1}{4\sqrt{27}} \frac{\text{in}^2}{\text{sec}}$
b. $x = 3$seconds
c. $y \approx 9.02$inches

5. Let $f(x) = xe^{x-1}, -4 \le x \le 4$.

a. On what intervals is $f(x)$ increasing? Decreasing?

b. On what intervals is $f(x)$ concave up? Down?

c. Find the absolute maximum and minimum values of $f(x)$, and the x values where they occur. Hint: there's a slightly subtle comparison of two numbers. To deal with this, it might help either to look at the first derivative, or to remember that e is greater than 2 – I recommend the first one.
Answers: **a.** Increasing on (-1,4], decreasing on [-4,-1)
b. Concave up on (-2,4], concave down on [-4,-2)
c. Absolute minimum value of $f(x)$ is $-e^{-2}$, occuring at $x = -1$; absolute maximum value of $f(x)$ is $4e^3$ occuring at $x = 4$

6. a. State the Mean Value Theorem.

b. Let $f(x)$ be a function that is continuous on the interval [1,5], differentiable on the interval (1,5), and satisfies

$$f'(x) = 5 + \sin(g(x))$$

for some function $g(x)$. If $f(1) = 10$, use the Mean Value Theorem to explain why it is impossible for $f(5)$ to equal 50. (Hint: what is the greatest possible number that $f'(x)$ could ever be? What is the least possible number of $f'(x)$? Even though you don't know what $g(x)$ is, you should be able to answer these if you know basic facts about the sine function.)

Answers: a. See your textbook or notes.

b. If $f(5)$ were to equal 50, then since f is continuous and differentiable, according to the MVT there would be some c between 1 and 5 with $f'(c) = \frac{50-10}{5-1} = \frac{40}{4} = 10$. On the other hand, $\sin(g(x))$ is always between -1 and 1, so $f'(x)$ will always be between 4 and 6. Hence, $f'(c)$ could not equal 10 for any c. Thus, it is impossible for $f(5) = 50$.