Problem 3. Using the curvature formula

\[ \kappa(t) = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|^3} \]

compute the curvature of the particle's path at \( t = 3 \). Intuitively, what does the curvature measure?

\[ \vec{r}'(t) = \langle 2t, 3t^2, e^t \rangle, \quad \vec{r}''(t) = \langle 2, 6t, e^t \rangle. \]

\[ \vec{r}'(t) \times \vec{r}''(t) = \langle 3t^2(e^t) - e^t(6t), e^t(2) - 2t(e^t), 2t(6t) - 3t^2(2) \rangle \]

\[ = \langle e^t(3t^2 - 6t), e^t(2 - 2t), 6t^2 \rangle. \]

So

\[ \vec{r}'(3) \times \vec{r}''(3) = \langle e^3, 9, e^3(-4), 54 \rangle. \]

Thus

\[ K(3) = \frac{|\vec{r}' \times \vec{r}''(3)|}{|\vec{r}'(3)|^3} = \frac{\sqrt{316 + 16e^6 + 2916}}{(765 + e^6)^{3/2}} = \frac{\sqrt{2916 + 976e^6}}{(765 + e^6)^{3/2}} \]

This curvature measures how fast the curve is bending at time \( t = 3 \).

Problem 4. The acceleration of a particle is given by the equation \( \vec{a}(t) = (0, 1, \sin(t)) \). Its initial position is \( \vec{r}(0) = (0, 1, 2) \) and its initial velocity is \( \vec{v}(0) = (1, 1, 1) \). What is the equation \( \vec{r}(t) \) for its position?

Since the derivative of velocity is acceleration,

\[ \vec{v}(t) = \int \vec{a}(t) \, dt = \int \langle 0, 1, \sin t \rangle \, dt = \langle 1, t, -\cos t \rangle + \vec{C}. \]

\[ \vec{v}(0) = \langle 1, 1, 1 \rangle = \langle 1, 0, -1 \rangle + \vec{C}. \]

So \( \vec{C} = \langle 0, 1, 2 \rangle \).

\[ \vec{v}(t) = \langle 1, t, -\cos t \rangle + \langle 0, 1, 2 \rangle = \langle 1, 1 + t, 2 - \cos t \rangle. \]

Since the derivative of position is velocity,

\[ \vec{r}(t) = \int \vec{v}(t) \, dt = \int \langle 1, 1 + t, 2 - \cos t \rangle \, dt = \langle t, t + \frac{1}{2}t^2, 2t-\sin t \rangle + \vec{D}. \]

\[ \vec{r}(0) = \langle 0, 1, 2 \rangle = \langle 0, 0, 0 \rangle + \vec{D} \quad \Rightarrow \quad \vec{D} = \langle 0, 1, 2 \rangle. \]

So

\[ \vec{r}(t) = \langle t, 1 + t + \frac{1}{2}t^2, 2 + 2t - \sin t \rangle. \]