Each answer must be mathematically justified. Don’t forget your name.

Problem 1. Find
\[
\min x + y + 2z \text{ subject to } x^2 + y^2 = 1 \text{ and } x + z = 0.
\]

Problem 2. Find
\[
\min x^2 + y^2 + z^2 \text{ subject to } x^2 + (y - 8)^2 + (z - 6)^2 = 25.
\]
Provide a geometric interpretation.

Problem 4. The goal of this problem is to solve the minimization problem
\[(P): \quad \min 2x + y - z \text{ subject to } (x - 1)^2 + (x + y - z)^2 + z^2 \leq 2\]
We set \(f(x, y, z) = 2x + y - z\) and
\[
U = \{(x, y, z) : (x - 1)^2 + (x + y - z)^2 + z^2 < 2\}
\]
(a) Show that \(f\) has no minimum or maximum on \(U\). (Hint: first show that \(U\) is open.)
(b) Solve the minimization problem
\[
\min 2x + y - z \text{ subject to } (x - 1)^2 + (x + y - z)^2 + z^2 = 2
\]
(c) Solve the minimization problem (P). (Hint: use the extreme value theorem.)

Problem 4. Use the Kuhn–Tucker condition to solve the minimization problem
\[
\min x^3 + xy \text{ subject to } x \geq 1 \text{ and } y \geq 1.
\]

Problem 5. (a) Use the Kuhn–Tucker condition to show that a necessary condition for \((x, y)\) to solve
\[(P): \quad \max x^3 - 3x - y \text{ subject to } x + y \leq 1, \ y \geq 0
\]
is \((x, y) = (-1, 0)\).
(b) Is the corresponding Lagrangian concave?
(c) Can you use the extreme value theorem as in Problem 3(c) to solve (P)?
(d) Find the global maximum of the function \(g(x) = x^3 - 3x\) defined on \((-\infty, 1]\) and conclude that \((-1, 0)\) solves the maximization problem (P).