HW3 - Due 02/06

Each answer must be mathematically justified. Don’t forget your name.

Problem 1. Find a $2 \times 2$ matrix $B$ such that $B^3 = A$, where

\[ A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix} \]

Problem 2. Is there a $3 \times 3$ symmetric matrix $A$ with distinct eigenvalues and eigenvectors

\[ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \]?

Problem 3. Test whether these quadratic forms are positive definite or negative definite:

(a) $q_1(x_1, x_2, x_3) = 10x_1^2 + 10x_2^2 + 10x_3^2 + 2x_1x_2 + 2x_2x_3 + 2x_3x_1$,
(b) $q_2(x_1, x_2, x_3) = x_1^2 + 2x_2^2 - 4x_3^2 + 2x_1x_2 - 2x_2x_3 - 2x_3x_1$.

Problem 4. Write the quadratic form

\[ q(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + 6x_3^2 + 2x_1x_2 + 2x_1x_3 + 6x_2x_3 \]

as a sum of squares. Deduce that $q$ is positive definite.

Problem 5. Find the equation of the tangent plane of the graphs of the function

(a) $f(x, y) = y^3 - 2y + x$ at $(x, y) = (1, 1)$;  (b) $g(x, y, z) = xz + 2y^2z^2$ at $(x, y, z) = (-1, 1, 0)$.

Problem 6. Find all local minimum and maximum of the following functions:

(a) $f(x, y) = x^2 + 2x + 2y + 2y^2 + 2xy + 1$;  (b) $g(x, y) = 4x^2 + 4xy + 2y^2 - 3$;
(c) $h(x, y) = 3x + 3xy + y^3$.

Problem 7. Let $f(x, y) = x^2 + 2xy + y^3 + x^3$.

(a) Show that $f$ has a saddle point at $(0, 0)$.
(b) Find some numbers $a$ and $b$ such that the function $g(t) = f(at, bt)$ has a local minimum at $t = 0$.
(c) Find some numbers $c$ and $d$ such that the function $h(t) = f(ct, dt)$ has a local maximum at $t = 0$. 

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