Algebraic Topology II, Problem Set 7

Due Wednesday, March 12, 2008

1. [Hatcher, §2.3, Exercise 4] Show that the wedge axiom for homology theories follows from the other axioms in the case of finite wedge sums.

2. [Hatcher, §4.3, Exercise 1] Show there is a map $\mathbb{RP}^\infty \to \mathbb{CP}^\infty$ which induces the trivial map on $H_*(\cdot;\mathbb{Z})$ and a nontrivial map on $H^*(\cdot;\mathbb{Z})$. How is this consistent with the universal coefficient theorem? [Recall that $\mathbb{CP}^\infty$ is a $K(\mathbb{Z},2)$.]

3. [Hatcher, §4.3, Exercise 4] Given abelian groups G and H, show that the space of maps $\langle K(G,n), K(H,n)\rangle$ is isomorphic to $\mathrm{Hom}(G,H)$ via the map sending a homotopy class $[f]$ to the induced isomorphism f_* on π_n.

4. [Hatcher, §4.F, Exercise 1] Show that the first two axioms of a homology theory and the direct limit axiom imply the wedge sum axiom. Show the first two axioms and the wedge sum axiom imple the direct limit axiom for countable CW complexes.

5. [Hatcher, §4.F, Exercise 3] Show that for any sequence of basepoint-preserving maps $Z_1 \to Z_2 \to \cdots$, the natural map $\varinjlim \Omega Z_n \to \Omega \varinjlim Z_n$ is a weak homotopy equivalence, where the direct limits mean mapping telescopes.