Algebraic Topology II, Problem Set 6

Due Wednesday, March 5, 2008

1. (a) Let $i\colon A \to B$ be an inclusion, and let F_i be its homotopy fiber. In class I asserted that $\pi_n(F_i) \cong \pi_{n+1}(B, A)$. Verify this assertion.

   (b) Let $F \hookrightarrow E \to B$ be the three terms of a fibration. Show that $\Omega F \to \Omega B \to F$ is also homotopy equivalent to the three terms of a fibration, with the map from $\Omega B \to F$ we defined in class.

2. Hatcher, §4.3, Exercise 8

3. Hatcher, §4.3, Exercise 9

4. Hatcher, §4.3, Exercise 10

5. Hatcher, §4.H, Exercise 2

6. Do Hatcher, §4.H, Exercise 3. Use this to conclude that the homotopy fiber E_f of a map $f\colon A \to B$ is uniquely characterized up to fiber homotopy equivalence by the requirements that the map $A \to E_f$ be a homotopy equivalence and the map $E_f \to B$ be a fibration.