PRACTICE FINAL

PROFESSOR THURSTON

To receive full credit, you must explain your answers.

No calculators of any type are allowed.

(1) Let P_1, P_2, P_3 be the planes defined by the equations below.

$$P_1 : x + y = 1$$
$$P_2 : y - z = 2$$
$$P_3 : y + z = -1$$

  (a) Find the line of intersection between P_1 and P_2.
  (b) Find the point of intersection between the line you found in part (a) and P_3.
  (c) Find the z coordinate of the same point (the point of intersection of P_1, P_2, and P_3) using Cramer's rule.

(2) Let $\mathbf{v} = \langle 1, 1, 0\rangle$, and let $\mathbf{w}$ be a vector of length 3 at $60°$ counterclocwise from $\mathbf{v}$, also in the xy plane.
  (a) What is $\mathbf{v} \cdot \mathbf{w}$?
  (b) What is $\mathbf{v} \times \mathbf{w}$?
  (c) What is $\mathbf{w}$?

(3) Let $f(x, y) = xe^{x+y}$.
  (a) Let $\mathbf{u} = \langle 1/\sqrt{2}, 1/\sqrt{2}\rangle$. Find the directional derivative $D_{\mathbf{u}}f(1, -1)$.
  (b) Find $\nabla f(1, -1)$.
  (c) Find the tangent plane to the graph of f at $(x, y) = (1, -1)$.
  (d) Use the linear approximation to f near $(1, -1)$ to approximate $f(1.1, -1.2)$.

(4) (a) Find the tangent plane to the surface $z^2 = x^2 + 2y^2 + 3$ at the point $(2, 1, 3)$.
  (b) If $z = xy^3 + x^4 + xy$ and $x = t + 1$, $y = t^2$, find dz/dt at $t = 1$.

(5) Find the critical points of the function

$$f(x, y) = (x^3 - 3x)e^{-y^2}$$

and classify whether they are local minimima, maxima, or saddle points.

(6) Find the largest box with one vertex at $(0, 0, 0)$, the opposite vertex on the ellipsoid $E = \{(x, y, z)|x^2 + 4y^2 + 9z^2 = 1\}$ and with sides aligned with the x, y, and z axes. Use Lagrange multipliers.

(7) (a) Compute $(-1 + i)^4$.
  (b) Find all 4th roots of -4.

(8) Show that absolute value is multiplicative on complex numbers:

$$|z_1 z_2| = |z_1||z_2|$$

for z_1 and z_2 complex numbers.

(9) (a) Find the general solution to $6y' + 9y = 0$.
(b) Find the general solution to $y'' + 6y' + 9y = 0$.
(c) Find the solution to $y'' + 6y' + 9y = 0$ where $y(0) = 0$ and $y'(0) = 1$.
(d) Find the general solution to $y'' + 6y' + 9y = x^2 + e^x$.