

Calculus III Midterm 1 Sample Problems

Section 4, Prof. Dylan Thurston

1. Find the cosine of the angle between a diagonal of a cube (i.e., a line joining opposite vertices through the center of the cube) and one of its edges.
2. For $P = (1, 2, 1)$, $Q = (-1, 3, 2)$, and $R = (1, 1, 1)$ compute:
 - (a) The area of the triangle PQR ;
 - (b) Two unit vectors perpendicular to the plane through the points P , Q , and R ;
 - (c) An equation of the plane passing through the points P , Q , and R .
3. Identify and sketch the graphs of the surfaces below. (On the test, a set of sample graphs will be provided for you to choose from.)
 - (a) $z = -\sqrt{1 - x^2}$
 - (b) $x^2 + 4y^2 + z^2 = 2x - 4z - 1$.
4. Write down parametric equations describing the ellipse $x^2/9 + y^2/4 = 1$ so that
 - (a) the curve is traced clockwise
 - (b) the curve is traced counterclockwise.
5. True or False?
 - (a) The two sets of parametric equations $x = t, y = t^2 + 1$ and $x = 3t, y = 9t^2 + 1$ correspond to the same implicit (or Cartesian) equation.
 - (b) The graph of the parametric equations $x = t^2$ and $y = t^2$ is the line $y = x$.
 - (c) If y is a function of t and x is a function of t , then y is a function of x .
 - (d) If $f(t_1) = 0$ and $g(t_2) = 0$, then the curve represented by the parametric equations $x = f(t)$ and $y = g(t)$ passes through the origin.
6. Let $P = (1, 2, 3)$ and $Q = (4, 5, 9)$.
 - (a) Find the distance between P and Q .
 - (b) Find a unit vector from the point P and toward the point Q .
 - (c) Find a vector of length 16 pointing in the opposite direction of \overrightarrow{PQ} .
 - (d) Find a point R such that \overrightarrow{PR} is a vector of length 16 pointing in the opposite direction of \overrightarrow{PQ} .
 - (e) Find the midpoint of the line segment from P to Q .

7. Write down parametric equations describing the line that passes through $(1, 2)$ and $(5, -2)$.
8. Let $\vec{a} = \langle 1, 2 \rangle$ and $\vec{b} = \langle -1, 3 \rangle$. Find the following vectors and illustrate them geometrically.
- (a) $\vec{a} + \vec{b}$ (b) $-\vec{a} + \vec{b}$ (c) $2\vec{a}$ (d) $-3\vec{b}$ (e) $2\vec{a} - 3\vec{b}$
9. Which pairs from the following list of vectors are perpendicular to one another?
 $\vec{u} = (1, \sqrt{3}, 0)$, $\vec{v} = (1, 0, \sqrt{3})$, and $\vec{w} = (\sqrt{3}, 1, -1)$.
10. Write $\vec{V} = (3, 2, -6)$ as the sum of two vectors, one parallel and one perpendicular to $\vec{W} = (2, -4, 1)$.
11. Show that the vectors $(\vec{b} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{c})\vec{b}$ and \vec{c} are perpendicular.
12. Let $\vec{a} = \langle 1, 2 \rangle$ and $\vec{b} = \langle 1, -1 \rangle$. Use Cramer's Rule to find numbers r and s so that $\langle 2, 1 \rangle = r\vec{a} + s\vec{b}$.
13. Use vectors to prove that the diagonals of a rhombus (a parallelogram such that all four sides are the same length) are perpendicular.
14. What is known about θ , the angle between two non-zero vectors \vec{a} and \vec{b} , if
- (a) $\vec{a} \cdot \vec{b} = 0$?
(b) $\vec{a} \cdot \vec{b} > 0$?
15. Prove that $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$.
16. Prove that $|\vec{a} \cdot \vec{b}| \leq |\vec{a}| |\vec{b}|$. (This is known as the Cauchy-Schwartz inequality.)
17. Find the volume of the parallelepiped determined by the three vectors $\langle 1, 3, 1 \rangle$, $\langle 0, 5, 5 \rangle$, and $\langle 4, 0, 4 \rangle$.
18. Determine which points lie on the line which passes through the point $(-2, 3, 1)$ and is parallel to the vector $\vec{a} = \langle 4, 0, -1 \rangle$.
- (a) $(-6, 3, 2)$
(b) $(2, 1, 0)$
(c) $(6, 3, -2)$
19. Find the distance between the line $x = 2t - 1, y = 3t + 1$ and the point $(2, 2)$.

20. Find the point of intersection and the angle of intersection of the lines

$$L_1 : x = 4t + 2, y = 3, z = -t + 1$$

$$L_2 : x = 2s + 2, y = 2s + 3, z = s + 1$$

21. Find a set of parametric equations for the line of intersection of the planes $3x + 2y - z = 7$ and $x - 4y + 2z = 0$. Find the (acute) angle between these two planes.

22. (a) Find the distance between the point $(1, 2, 3)$ and the plane $2x - 2y + z = 7$.
(b) Find the equation of the line L that goes through $(1, 2, 3)$ and perpendicular to the plane $2x - 2y + z = 7$.
(c) Find the point P of intersection between the line L from (b) and the plane $2x - 2y + z = 7$.
(d) Find the distance between the point $(1, 2, 3)$ and the point P from (c). Can you interpret this result geometrically?

23. Find the distance between the planes $x - 3y + 4z = 10$ and $x - 3y + 4z = 6$.

24. Find and plot all the eighth roots of $256 = 2^8$ in the complex plane.

25. Expand out $(a + bi)^3$ and collect the real and imaginary parts. Use this to express $\sin(3\theta)$ and $\cos(3\theta)$ in terms of $\sin \theta$ and $\cos \theta$.