Problem Set #9
Section 4, Prof. Dylan Thurston
Due Wednesday, November 23, 2005

Please complete and hand in the exercises. The exercises will not be graded.

Note that Thanksgiving is November 24. If you will not be in class on November 23, it is your responsibility to get me the problem set before the due date. Some ways to hand it in include:

- Hand it in on Monday.
- Put it in the homework return bin, on the fourth floor of Mathematics Hall.
- Put it under my door, Mathematics 614.
- E-mail me a Postscript or PDF file.

1 Exercises

1. Section 14.5, exercises 3, 7, 21, 35.
2. Section 14.6, exercises 1, 7, 11, 21, 39.

2 Homework

1. (a) Consider the function $f(x,y) = x^2y + xy^2$ along the line $x = 1 + t$, $y = 1 + 2t$. Find df/dt and $d^2f/(dt)^2$.

   (b) Now do part (a) generically: for any function $f(x,y)$ along the line $x = x_0 + at$, $y = y_0 + bt$, find df/dt and $d^2f/(dt)^2$ in terms of the partial derivatives of f and the given constants. Simplify your answer.

2. Consider the topographical map on the next page, which is the same map from homework 8. There is a horizontal scale at the bottom; the vertical scale is marked in feet, with one contour line every 10 feet and one thick line every 50 feet.

   (a) Estimate the distance between points A and B (in miles) and the difference in height between A and B. Use this to estimate the directional derivative at point A in the direction of point B. What are the units of this derivative?

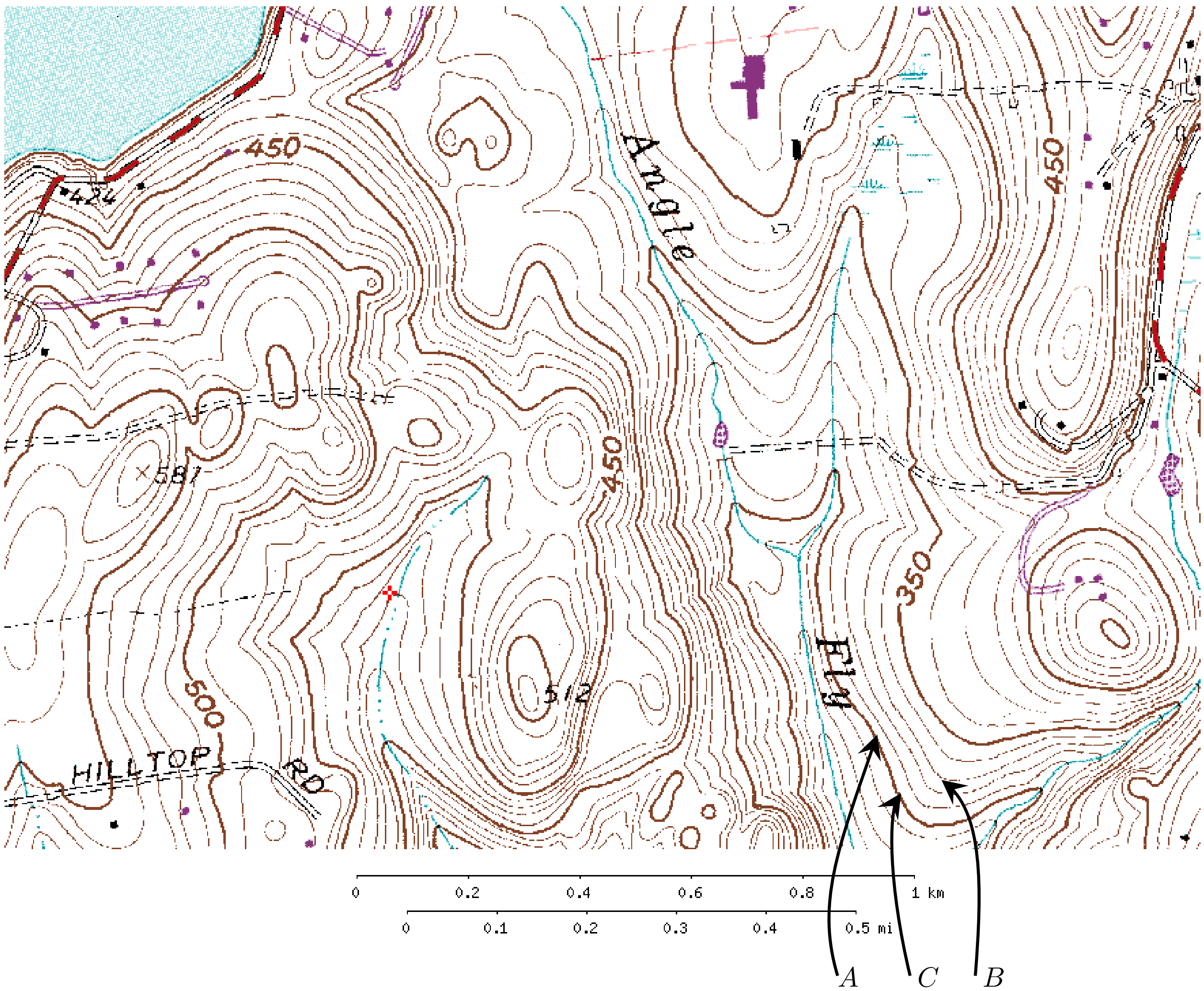
   (b) What is the directional derivative at point A in the direction of point C?

   (c) Estimate the gradient vector at point A: draw a vector in the correct direction on the graph, and estimate the length.

3. Show that a differentiable function f decreases most rapidly at $\vec{x}$ in the direction opposite to the gradient vector, that is, in the direction of $-\vec{\nabla} f(\vec{x})$.

4. (a) Consider the surface defined by $z^2 - x^2 - y^2 = -1$. Find the tangent planes at the points $(1,0,0)$ and $(-1,2,2)$.

   (b) Find the intersection between this same surface and its tangent plane at $(1,0,0)$. You should get two lines. In fact, this surface is a *ruled surface*: its intersection with its tangent plane at any point consists of two lines.


450
424
Angle
450
581
450
512
500
HILLTOP
RD
Fly
350
0
0.2
0.4
0.6
0.8
1 km
0
0.1
0.2
0.3
0.4
0.5 mi
A
C
B