Problem Set #4

Section 4, Prof. Dylan Thurston

Due Wednesday, October 5, 2005

As a reminder, you are encouraged to work with other students, but please write up your solutions yourself and credit your collaborators.

1 Homework

1. Find the dihedral angle of the tetrahedron. This is the exterior angle between two adjacent faces of the tetrahedron. Recall that you can take the vertices of the tetrahedron to be $(1,1,1)$, $(1,-1,-1)$, $(-1,1,-1)$, and $(-1,-1,1)$. Remember that there is some ambiguity in finding the dihedral angle between planes; in this case, the fact that the dihedral angle of the tetrahedron is bigger than 270° removes the ambiguity.

2. Let L_1 be the line $\vec{r} = \langle 1,1,0\rangle + t\langle -2,1,0\rangle$ and let L_2 be the line $\vec{r} = \langle -1,0,-1\rangle + t\langle 0,1,-1\rangle$

   (a) Find the plane containing L_1 and parallel to L_2. Verify that this plane does not contain L_2; L_1 and L_2 are said to be *skew lines*.

   (b) Find the distance between the plane you found in part (a) and a point on L_2. Does the distance depend on the point on L_2 that you picked? In fact, this distance is the closest distance between the lines L_1 and L_2.

3. Use vectors and the cross product to give a parametric equation for the line of intersection between the planes $x+y+z=1$ and $x+y-z=1$.

4. Find the traces (sections) of the surface $x+y^2+z^2=0$ parallel to the xy, yz, and xz planes. Sketch the resulting surface.

5. Evaluate $(1+\sqrt{3}i)^3$. Plot $(1+\sqrt{3}i)$, $(1+\sqrt{3}i)^2$, and $(1+\sqrt{3}i)^3$ in the complex plane.

2 Exercises

(These problems are not to be handed in; they are for your practice.)

1. Section 12.5: Exercises 45, 47, 57, 61, 65, 70 (in the 4th ed.: Exercises 41, 43, 53, 57, 61, 66).

2. Section 12.6: Exercises 10, 20, 21–28 (in the 4th ed.: look at a 5th ed. book for exercise 10).

3. Homework 1 and 2 from the handout on complex numbers.