1.3

4: The rref form of the matrix is \[
\begin{pmatrix}
1 & 0 & -1 \\
0 & 1 & 2 \\
0 & 0 & 0
\end{pmatrix}
\]. Hence the rank is 2.

18: \(\text{Ans} = \begin{pmatrix} 5 \\ 11 \\ 17 \end{pmatrix}\).

24: If \(A\vec{x} = \vec{b}\) has a unique solution then matrix \(A\) must be full rank. In this case for any \(\vec{c}\), the equation \(A\vec{x} = \vec{c}\) has a unique solution as well.

28: If rank of a \(5 \times 3\) matrix is three then the two bottom rows must be zero:
\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

2.1

6: \[
\begin{pmatrix}
1 & 4 \\
2 & 5 \\
3 & 6
\end{pmatrix}
\]

8: \[
\begin{align*}
x_1 + 7x_2 &= y_1 \\
3x_1 + 20x_2 &= y_2
\end{align*}
\]
\[
\begin{align*}
-3x_1 &= y_1 \\
-7x_2 &= y_2
\end{align*}
\]

Therefore the inverse is \[
\begin{pmatrix}
-20 & 7 \\
3 & -1
\end{pmatrix}
\]

42: 

b) The image of \[
\begin{pmatrix}
1 \\
1/2 \\
1/2
\end{pmatrix}
\] is \((0, 0)\).

c) The points which are mapped to \((0, 0)\) are precisely the scalar multiples of \[
\begin{pmatrix}
1 \\
1/2 \\
1/2
\end{pmatrix}
\].
50: a) Suppose there was $a$ grams of the platinum alloy and $b$ grams of the silver alloy in the crown. Solve the equations: $a + b = 5000$, $a/20 + b/10 = 370$. The solution is: $a = 2600$ and $b = 2400$.

b) Write the above equations in a matrix: \[
\begin{pmatrix}
1 & 1 \\
1/20 & 1/10
\end{pmatrix}.
\]

c) The determinant of the above matrix is 20. Therefore it’s invertible. The inverse is \[
\begin{pmatrix}
2 & -20 \\
-1 & 20
\end{pmatrix}.
\]

2.2

2: \[
\begin{pmatrix}
1/2 & \sqrt{3}/2 \\
\sqrt{3}/2 & -1/2
\end{pmatrix}
\]

4: The matrix represents clockwise rotation by $45^\circ$ followed by scaling by a factor of $\sqrt{2}$.

26: a) \[
\begin{pmatrix}
4 & 0 \\
0 & 4
\end{pmatrix}
\]

b) \[
\begin{pmatrix}
1 & 0 \\
0 & 0
\end{pmatrix}
\]

c) \[
\begin{pmatrix}
4/5 & 3/5 \\
-3/5 & 4/5
\end{pmatrix}
\]

d) \[
\begin{pmatrix}
1 & 2 \\
0 & 1
\end{pmatrix}
\]

e) \[
\begin{pmatrix}
-4/5 & 3/5 \\
3/5 & 4/5
\end{pmatrix}
\]

32: a) The matrix transformation rotates a vector through angle $\alpha$. Vector $\vec{v}$ is a unit vector which makes angle $\beta$ with the positive x-axis. Therefore $D\vec{v}$ would be at an angle of $\alpha + \beta$.

b) Compare the entries of $D\vec{v}$ with \[
\begin{pmatrix}
\cos(\alpha + \beta) \\
\sin(\alpha + \beta)
\end{pmatrix}.
\]
This proves $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$, $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$.,