

Consider a principle $SU(2)$ -bundle P over a compact 4-manifold M and a $W^{1,2}$ -connection A of P satisfying $\|FA\|_{L^2(M)} \leq \Lambda$. Our main result is the existence of a global section $\sigma : M \rightarrow P$ with controllably many singularities such that the connection form σ^*A satisfies the Coulomb equation $d^*(\sigma^*A) = 0$ and moreover admits a sharp estimate.

In this talk, we first recall some preliminaries and then outlines the proof. If time allows we shall elaborate on the ideas to overcome the main difficulties in this problem, which include an ϵ -regularity theorem for the Coulomb gauge, an annular-bubble regions decomposition for the curvature, and studying the singularity behavior of the Coulomb gauge on each annular and bubble region.