We prove that any manifold diffeomorphic to S^3 and endowed with a generic metric contains at least two embedded minimal two-spheres. The existence of at least one minimal two-sphere was obtained by Simon-Smith in 1983. Our approach combines ideas from min-max theory and mean curvature flow. We also establish the existence of smooth mean convex foliations in three-manifolds. Finally, we apply our methods to solve a problem posed by S.T. Yau in 1987, and to show that the assumptions in the multiplicity one conjecture and the equidistribution of widths conjecture are in a certain sense sharp. This is joint work with Dan Ketover.