

The Lojasiewicz inequalities for real analytic functions on Euclidean space were first proved by Stanislaw Lojasiewicz during the early 1960s using methods of semianalytic and subanalytic sets, arguments later simplified by Bierstone and Milman (1988 and 1997). The primary reference due to Lojasiewicz for proofs of his inequalities was circulated in 1965 as an IHES preprint, but was never published. In this talk, we first describe elementary geometric or coordinate-based proofs of the Lojasiewicz inequalities in special cases where the function is (a)  $C^2$  and MorseBott, or (b)  $C^N$  and generalized MorseBott, or (c)  $C^1$  with simple normal crossings. We then show, partly following Bierstone and Milman (1997) and using resolution of singularities for real analytic varieties, that the gradient inequality for an arbitrary real analytic function follows from the special case where it has simple normal crossings. Finally, we show that a real analytic function has the optimal Lojasiewicz exponent  $1/2$  if and only if it is Morse-Bott, and so its critical set is a smooth submanifold. This talk is based in part on arXiv:1708.09775.