HOMEWORK 9

6.1.4. We look for a solution $u = u(r)$. Thus u must satisfy

$$u_{rr} + \frac{2}{r}u_r = 0 \Rightarrow (r^2 u_r)_r = 0 \Rightarrow r^2 u_r = c_1 \Rightarrow u = -\frac{c_1}{r} + c_2.$$

From the boundary conditions

$$u(a) = A \Rightarrow -\frac{c_1}{a} + c_2 = A$$

$$u(b) = B \Rightarrow -\frac{c_1}{b} + c_2 = B.$$

Solving for c_1, c_2 we obtain

$$c_1 = (B - A)(1/a - 1/b)^{-1}, \quad c_2 = B + (B - A)(1/a - 1/b)^{-1}/b.$$

6.1.11. Integrate $\Delta u = f$ to obtain

$$(1) \qquad \int_D \Delta u d\mathbf{x} = \int_D f d\mathbf{x}.$$

From the divergence theorem

$$(2) \qquad \int_D \Delta u d\mathbf{x} = \int_D \operatorname{div}(\nabla u) d\mathbf{x} = \int_{\partial D} \nabla u \cdot \mathbf{n} dS = \int_{\partial D} \frac{\partial u}{\partial n} dS.$$

Combining (1), (2) and the boundary condition $\frac{\partial u}{\partial n} = g$ on ∂D, we obtain the desired equality.