A classical problem in the calculus of variations is to determine the regularity of Lipschitz minimizers of $\int F(\nabla u)$, where $F$ is convex. When $F$ is smooth and uniformly convex, De Giorgi and Nash showed that minimizers are smooth. If the graph of $F$ contains a line segment, minimizers are no better than Lipschitz. In the intermediate case that $F$ is strictly convex but its second derivatives tend to zero or infinity on some set (which arises in many applications), it is reasonable to ask whether Lipschitz minimizers are $C^1$. We will discuss recent results that answer this question positively in some cases and negatively in general, and highlight a connection between this problem and classical differential geometry.