In this talk, based on joint work with Plamen Stefanov and Gunther Uhlmann, I discuss the boundary rigidity problem on manifolds with boundary (for instance, a domain in Euclidean space with a perturbed metric), i.e. determining a Riemannian metric from the restriction of its distance function to the boundary. This corresponds to travel time tomography, i.e. finding the Riemannian metric from the time it takes for solutions of the corresponding wave equation to travel between boundary points.

This non-linear problem in turn builds on the geodesic X-ray transform on such a Riemannian manifold with boundary. The geodesic X-ray transform on functions associates to a function its integral along geodesic curves, so for instance in domains in Euclidean space along straight lines. The X-ray transform on symmetric tensors is similar, but one integrates the tensor contracted with the tangent vector of the geodesics. I will explain how, under a convexity assumption on the boundary, one can invert the local geodesic X-ray transform on functions, i.e. determine the function from its X-ray transform, in a stable manner. I will also explain how the analogous result can be achieved on one forms and 2-tensors up to the natural obstacle, namely potential tensors (forms which are differentials of functions vanishing at the boundary, respectively tensors which are symmetric gradients of one-forms vanishing at the boundary).

Here the local transform means that one would like to recover a function (or tensor) in a suitable neighborhood of a point on the boundary of the manifold given its integral along geodesic segments that stay in this neighborhood (i.e. with both endpoints on the boundary of the manifold). Our method relies on microlocal analysis, in a form that was introduced by Melrose.