The goal of the talk is to discuss a proof of the Levy-Gromov inequality for metric measure spaces (joint with Cavalletti), a quantitative version of the Levy-Gromov isoperimetric inequality (joint with Cavalletti and Maggi) as well as other geometric/functional inequalities (joint with Cavalletti and Semola). Given a closed Riemannian manifold with strictly positive Ricci tensor, one estimates the measure of the symmetric difference of a set with a metric ball with the deficit in the Levy-Gromov inequality. The results are obtained via a quantitative analysis based on the localisation method via L1-optimal transport.