Calculus 1 Assignment 3
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1 Rock Throwing

Problem 1.1: Alice drops a rock from a height of 5 meters in an alternate universe. In this alternate universe, the height of the rock above the ground is given by \( h(t) = 5 - 5t^3 \). How fast is the rock falling when it hits the ground? Only use arguments similar to those discussed in class and in the first two chapters of Stewart.

2 Limits

Problem 2.1: What are the following limits?
   a) \( \lim_{x \to 0} \frac{\sin(\pi x)}{x} \)
   b) \( \lim_{x \to 0} \frac{\log(ax+1)}{x} \), where \( a \) is an arbitrary real number. (Hint: experiment with some fixed values of \( a \).)
   c) \( \lim_{x \to 0} x \sin\left(\frac{1}{x}\right) \)
   d) \( \lim_{x \to 0} \frac{x^{\frac{1}{x+1}}}{} \)
   e) \( \lim_{x \to 1} f(x) \), where \( f(x) = 0 \) if \( x \notin \mathbb{Q} \), and \( f(x) = 1 \) if \( x \in \mathbb{Q} \).
   f\(^*\) \( \lim_{x \to 0.5} f(x) \), where \( f(x) = 0 \) if \( x \notin \mathbb{Q} \), and \( f(x) = \frac{1}{q} \) if \( x = \frac{p}{q} \) is a fraction in lowest terms and \( q > 0 \).

2.2: Give an example of a function \( f \) and a point \( a \) such that \( f(a), \lim_{x \to a^+} f(x) \), and \( \lim_{x \to a^-} f(x) \) all exist and are all unequal.

2.3: Suppose \( f \) and \( g \) are “nice” functions. Show that
\[
\lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} = f(x) \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} + g(x) \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]
What assumptions did you have to make about \( f \) and \( g \) (i.e. what does “nice” mean here?)
(Hint: Add and subtract \( \lim_{h \to 0} \frac{f(x+h)g(x)}{h} \) to the left hand side.)