Knots and 3-manifolds: Problem Set 5
due Tuesday, July 30

1. (a) Let M be a three-manifold that admits a Heegaard splitting of genus g. Prove that $\pi_1(M)$ has a presentation with g generators and g relations.

(b) Prove that $\pi_1(L(p,q)) = \mathbb{Z}/p$ by applying the Seifert-van Kampen theorem to the Heegaard decomposition of $L(p,q)$ given in class.

2. Classify T^2-fibrations over S^1 and compute their fundamental groups. Hint: Given such a fibration E, consider a neighborhood N of the union of a fiber and an arc going around the base. Then apply the Seifert-van Kampen theorem to the decomposition of E into N and its complement.

3. (a) Consider the ball $B^3 = \{x \in \mathbb{R}^3 : |x| \leq 1\}$. If we identify every point x on the boundary S^2 of B^3 with $-x$, the resulting space is a closed three-dimensional manifold M. Show that M is diffeomorphic to $\mathbb{RP}^3$.

(b) Prove that $UT(S^2)$ is diffeomorphic to $\mathbb{RP}^3$.