Recall
\[ A \text{ a full subset of } \mathbb{Z}^n \]
\[ P = \text{convex hull}(A) \]
\[ \nu : A \to \mathbb{R} \]
\[ \nu \text{ maximal convex extension of } \nu \text{ to } P \]
\[ \overline{\nu} \text{ gives a subdivision of } P \text{ with vertices are } A \]
this called a coherent subdivision

Setting
\[ X \text{ is complete toric variety defined by a fan } \Delta \]
\[ A = \{0, \nu \} \]
\[ \text{primitive vertices vertices of on rays in } \Delta \]
\[ \mathcal{C}_1, \mathcal{T}_1, A : \text{five pts} \]

Each maximal cone to be on \( \text{conv} \{0, \nu \in \mathcal{C}\} \)
Assume each \( P(\mathcal{C}) \) a minimal simplex \( \iff \nu \in \mathcal{C} \) form basis
for \( \mathbb{Z} \iff X \text{ smooth} \)

Choose \( W(z) = -1 + \sum_{\mathbf{0} \neq \alpha \in A} z^\alpha \)
Lemma
If we have ample line bundles on $X$
$\Rightarrow$ a coherent subdivision of $P$ which is $P(\mathbb{C})$

Divisors
precisely integral linear function on $|\Delta|$ given by some Cartier divisor, smooth Weil divisor
If $D = \sum a_i D_i$

distinguished point = variety for maximal dimension
unique torus fixed pt.

semi-group map

map $S^1 \to C$

$\sigma$: core
$v \mapsto 1$ $v \in S^1$ orthogonal
$v \mapsto 0$ otherwise $\Rightarrow X(\mathbb{C})$

$|\psi_D(v_i)|^2 = a_i$

Fact $\psi_D$ convex $\Rightarrow$ $D$ ample

$|\psi_D|_{\psi_D}$ convex $\Rightarrow$ $D$ nef

$|\psi_D|_{\psi_D}$ strictly convex $\Rightarrow$ $\nu$ ample
choose $\psi$

$\phi | A \rightarrow$ subdivision of $P$

0 a vertex of this subdivision

Each polytope through 0 is $P(\tau)$

$\phi | A$ convex support of

is $\psi \rightarrow \phi$

choose a $P(\tau)$

$\Delta_A \psi = \phi$

since $\phi$ is strictly convex

$P(\tau) \subset \frac{S(\tau)}{polytope}$

in subdivision

If $P(\tau) \& S(\tau)$

$S(\tau)$ has an additional vertex at $P(\tau)$

but $\phi$ is subdivision, doesn't create new vertices.

$V \cap V$ for some, $V_i$ is $\cap M \cap T$

$\Omega = \text{Trop}(W)$

$= -1 + \sum_{0 \neq \theta \in A} x^\theta$

locus of non-smoothness of $L_V$
**Pf**

\[ L_v : R \rightarrow R \]

\[ L_v(u) = \max_{\alpha \in A} (\langle \alpha, u \rangle - \phi(\alpha)) \]

\[ \nu = \Phi |_{A} \]

Each face is dual to a unique polytope \((n-K)\) subdivision of \(D\)

\[ K=0 \Rightarrow \text{component of the complement} \]

\[ d \in A \text{ act as max of } L_v \text{ on the component } C_d \]

\[ 0 \rightarrow \text{ component of } \mathbb{R}^n - \Pi \]

The component at \(0\)

\[ \langle \nu, y \rangle \leq \phi(\nu) - R \]

Symplectic

Choose

\[ W \text{ on } (\mathbb{C}^\ast)^n \]

\[ w = \sum_{j=1}^{n} \frac{dz \wedge d\bar{z}}{2i |z_j|^2} \]

\[ (\mathbb{T}^n)^{\ast} \]

\[ V_j \text{ on } \mathbb{R}^n \]

\[ \mathbb{Z}_j = \mathbb{Z} \text{ for which one lattice spanned by } \{2 \pi \mathbb{Z}_j \}_{j=1}^{n} \]

Fact

\[ T^* \mathbb{R}^n / L \cong (\mathbb{C}^\ast)^n \]

\[ (u, v_j) \rightarrow e^{u + i\theta_j} \]

Inverse

\[ z_j \rightarrow (\log |z_j|, \text{arg}(z_j)) \]