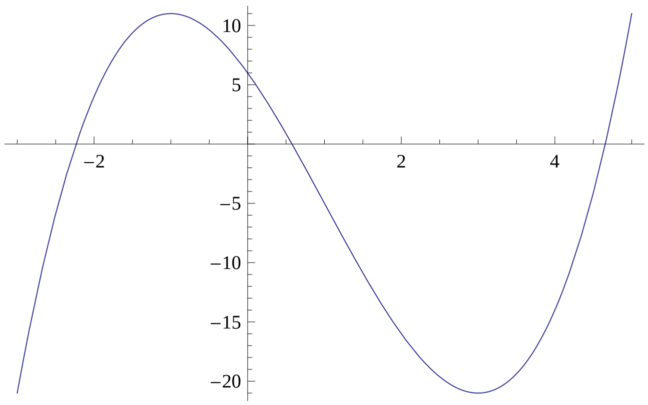
Problem 1.

FTFT FFFT

Problem 2.

(1) Increasing on $(-\infty, -1)$, $(3, +\infty)$. Decreasing on $(-1, 3)$.

(2) Concave up on $(1, +\infty)$. Concave down on $(-\infty, 1)$.

(3) 

(4) Absolute maximum $f(1) = -5$. Absolute minimum $f(3) = -21$.

Problem 3.

(1) $f'(x) = 8(3x^5 + x^2)^7 \cdot (15x^4 + 2x)$ (chain rule).

(2) $f'(x) = -8x \cdot e^{-4x^2}$ (chain rule).

(3) $f'(x) = -2(\ln x)^{-3} \cdot \dfrac{1}{x} = \dfrac{-2}{x(\ln x)^3}$ (chain rule).

(4) $f'(x) = \dfrac{1}{1 + (\sqrt{x})^2} \cdot \dfrac{1}{2}x^{-1/2} = \dfrac{1}{2\sqrt{x}(1 + x)}$ (chain rule).

Problem 4.

(1) Let $f(x) = x^{1/5}$. Its linear approximation at $a = 1$, is given by

$$L(x) = f'(a)(x - a) + f(a) = \frac{1}{5}(x - 1) + 1.$$

So

$$f(0.95) \approx L(0.95) = -0.01 + 1 = 0.99.$$

(2) By the chain rule, we have the relation

$$\frac{dS}{dt} = \frac{dS}{dr} \cdot \frac{dr}{dt}.$$

By assumption,

$$\frac{dS}{dt} = 8.$$

By the surface area formula for a sphere,

$$S = 4\pi r^2,$$

we know that

$$\frac{dS}{dr} = 8\pi r.$$

Thus

$$\frac{dr}{dt} = \frac{8}{8\pi r} = \frac{1}{\pi r} = \frac{1}{6\pi}.$$

(3) Rewrite

$$\frac{\sin x}{x^3} - \frac{1}{x^2} = \frac{\sin x - x}{x^3}.$$

Applying l'Hospital's rule three times we find that

$$\lim_{x\to 0} \frac{\sin x - x}{x^3} = \lim_{x\to 0} \frac{\cos x - 1}{3x^2} = \lim_{x\to 0} \frac{-\sin x}{6x} = \lim_{x\to 0} \frac{-\cos x}{6} = -\frac{1}{6}.$$

Problem 5.

By implicit differentiation we obtain

$$2x + 2y \cdot y' = 2(2x^2 + 2y^2 - x) \cdot (4x + 4y \cdot y' - 1).$$

Plugging in $(x, y) = (0, \frac{1}{2})$ gives

$$y' = 2 \cdot 1/2 \cdot (2y' - 1),$$

which simplifies to

$$y' = 2y' - 1.$$

Hence $y' = 1$. By the point-slope formula, the tangent line has equation

$$y = 1 \cdot (x - 0) + \frac{1}{2} = x + \frac{1}{2}.$$