Problem 1.

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Problem 2.

(1) Increasing on \((-\infty, -1), (3, +\infty)\). Decreasing on \((-1, 3)\).

(2) Concave up on \((1, +\infty)\). Concave down on \((-\infty, 1)\).

(3) Absolute maximum \(f(1) = -5\). Absolute minimum \(f(3) = -21\).

Problem 3.

(1) \(f'(x) = 8(3x^5 + x^2)^7 \cdot (15x^4 + 2x)\) (chain rule).

(2) \(f'(x) = -8x \cdot e^{-4x^2}\) (chain rule).

(3) \(f'(x) = -2(\ln x)^{-3} \cdot \frac{1}{x} = \frac{-2}{x(\ln x)^3}\) (chain rule).

(4) \(f'(x) = \frac{1}{1 + (\sqrt{x})^2} \cdot \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}(1 + x)}\) (chain rule).

Problem 4.

(1) Let \(f(x) = x^{1/5}\). Its linear approximation at \(a = 1\), is given by

\[ L(x) = f'(a)(x - a) + f(a) = \frac{1}{5}(x - 1) + 1. \]

So

\[ f(0.95) \approx L(0.95) = -0.01 + 1 = 0.99. \]

(2) By the chain rule, we have the relation

\[ \frac{dS}{dt} = \frac{dS}{dr} \cdot \frac{dr}{dt}. \]
By assumption, \[
\frac{dS}{dt} = 8.
\]
By the surface area formula for a sphere, 
\[
S = 4\pi r^2,
\]
we know that 
\[
\frac{dS}{dr} = 8\pi r.
\]
Thus 
\[
\frac{dr}{dt} = \frac{8}{8\pi r} = \frac{1}{\pi r} = \frac{1}{6\pi}.
\]

(3) Rewrite 
\[
\frac{\sin x}{x^3} - \frac{1}{x^2} = \frac{\sin x - x}{x^3}.
\]
Applying l’Hospital’s rule three times we find that 
\[
\lim_{x \to 0} \frac{\sin x - x}{x^3} = \lim_{x \to 0} \frac{\cos x - 1}{3x^2} = \lim_{x \to 0} \frac{-\sin x}{6x} = \lim_{x \to 0} \frac{-\cos x}{6} = -\frac{1}{6}.
\]

**Problem 5.**

By implicit differentiation we obtain 
\[
2x + 2y \cdot y' = 2(2x^2 + 2y^2 - x) \cdot (4x + 4y \cdot y' - 1).
\]
Plugging in \((x, y) = (0, \frac{1}{2})\) gives 
\[
y' = 2 \cdot 1/2 \cdot (2y' - 1),
\]
which simplifies to 
\[
y' = 2y' - 1.
\]
Hence \(y' = 1\). By the point-slope formula, the tangent line has equation 
\[
y = 1 \cdot (x - 0) + \frac{1}{2} = x + \frac{1}{2}.
\]