• You have 75 minutes to complete your work. The total: 40 points.
• Except for True/False and matching problems, show your reasons and computations.
• No notes, books, calculators, computers, or other electronic aids are allowed.

**Problem 1.** (8 points) Mark True or False. No justification is needed.

(1) T F For any function $f(x)$, it is always true that $\lim_{x \to a} f(x) = f(a)$.

(2) T F The inverse function of $f(x) = e^x$ is $f^{-1}(x) = e^{-x}$.

(3) T F $\cos(\arccos(\frac{3}{5})) = \frac{4}{5}$.

(4) T F Any one-to-one function passes the horizontal line test.

(5) T F $\lim_{x \to +\infty} \sin(x) = +\infty$.

(6) T F The derivative of the constant function $f(x) = e^5$ is $f'(x) = e^5$.

(7) T F $f(x) = \frac{1 + e^x}{1 - e^x}$ is an odd function.

(8) T F The equation $e^x = 2 + x$ has a solution in the interval $(0, 2)$.

**Problem 2.** (4 points) The graph of $y = f(x) = e^x$ is shown. Match the functions (1) – (5) on the left and curves (a) – (e) on the right. No justification is needed.

(1) $\leftrightarrow$ ( ) $y = 2f(x)$

(2) $\leftrightarrow$ ( ) $y = f(-x)$

(3) $\leftrightarrow$ ( ) $y = -f(x) + 6$

(4) $\leftrightarrow$ ( ) $y = f(2x + 2)$

(5) $\leftrightarrow$ ( ) $y = f(2x - 3)$

![Graph of $y = f(x) = e^x$ with curves a, b, c, d, e and matching functions]
Problem 3. (4 points) Match the graphs (1) – (5) on the top for \( y = f(x) \) with the graphs (a) – (e) on the bottom for the derivatives \( y = f'(x) \). No justification is needed.

Problem 4. (Limits) (3 × 3 = 9 points) Compute the following limits. Show your work. If the limit does not exist, explain why.

(1) \( \lim_{x \to 3} \frac{x^2 - 5x + 6}{x - 3} \).

(2) \( \lim_{x \to 0} \frac{|x|}{x} \).

(3) \( \lim_{x \to \infty} \frac{x}{\sqrt[4]{4x^2 + 1} + x} \).

Problem 5. (Derivatives) (3 × 3 = 9 points) Compute \( f'(x) \) for the following functions. Show your work.

(1) \( f(x) = 2x^5 - \frac{4}{x} \).

(2) \( f(x) = \frac{1 + x}{4 + e^x} \).

(3) \( f(x) = x^4 \cos(x) \).

Problem 6. (Tangent lines) (3 points) Draw a diagram to show that there are two tangent lines to the parabola \( y = x^2 \) that pass through the point \((0, -4)\). Find the coordinates of the points where these tangent lines intersect the parabola.

Problem 7. (Asymptotes) (3 points) Find all the horizontal and vertical asymptotes of the curve \( y = f(x) = \frac{x^2 - 1}{x^2 - 4} \).