Example 1. (see 16.6 Exercise 42) Let $S_1$ be the part of the cylinder $x^2 + z^2 = R^2$ that lies inside $y^2 + z^2 = R^2$. Find the area of $S_1$. 

Solution: We have $\text{Area}(S_1) = 8\text{Area}(S)$, where $S$ is the portion of $S_1$ in the first octant (see Figure 1).

![Figure 1](https://via.placeholder.com/150)

$S$ is given by 
\[ r(y, z) = \langle \sqrt{R^2 - z^2}, y, z \rangle, \quad (y, z) \in D \]
where $D = \{(y, z) \in \mathbb{R}^2 \mid y^2 + z^2 \leq R^2\}$.

\[ r_y = \langle 0, 1, 0 \rangle \]
\[ r_z = \langle -\frac{z}{\sqrt{R^2 - z^2}}, 0, 1 \rangle \]
\[ r_y \times r_z = \langle 1, 0, \frac{z}{\sqrt{R^2 - z^2}} \rangle \]
\[ |r_y \times r_z| = \sqrt{1 + \frac{z^2}{R^2 - z^2}} = \frac{R}{\sqrt{R^2 - z^2}} \]
\[ \text{Area}(S) = \int \int_D |r_y \times r_z| \, dA = \int \int_D \frac{R}{\sqrt{R^2 - z^2}} \, dA \]
\[ = \int_0^R \int_0^{\sqrt{R^2 - z^2}} \frac{R}{\sqrt{R^2 - z^2}} \, dy \, dz = \int_0^R Rdz = R^2. \]
So $\text{Area}(S_1) = 8R^2$.

Example 2. (see 15.6 Exercise 24) Let $S_2$ be the boundary of the solid that lies inside both cylinders $x^2 + z^2 = R^2$ and $y^2 + z^2 = R^2$. Find the area of $S_2$.

Solution: Let $S_1$ be the surface in Example 1. Then 
\[ \text{Area}S_2 = 2\text{Area}(S_1) = 16R^2. \]