Problem 1  Consider the integral
\[ \int_1^2 \int_x^2 12x \, dy \, dx + \int_2^4 \int_x^4 12x \, dy \, dx \]
(a) Sketch the region of integration.
(b) Reverse the order of integration and evaluate the integral that you get.

Problem 2  Consider the transformation of \( \mathbb{R}^2 \) defined by the equations given by \( x = u/v, \ y = v \).
(a) Find the Jacobian \( \frac{\partial(x,y)}{\partial(u,v)} \) of the transformation.
(b) Let \( R \) be the region in the first quadrant bounded by the lines \( y = x, \ y = 2x \) and the hyperbolas \( xy = 1, \ xy = 2 \). Sketch the region \( S \) in the \( uv \)-plane corresponding to \( R \).
(c) Evaluate \( \iint_R y^4 \, dA \).

Problem 3  Let \( S \) be the boundary of the solid bounded by the paraboloid \( z = x^2 + y^2 \) and the plane \( z = 4 \), with outward orientation.
(a) Find the surface area of \( S \). Note that the surface \( S \) consists of a portion of the paraboloid \( z = x^2 + y^2 \) and a portion of the plane \( z = 4 \).
(b) Use the Divergence Theorem to calculate the surface integral \( \iint_S F \cdot d\mathbf{S} \), where \( F = (x + y^2z^2)\mathbf{i} + (y + z^2x^2)\mathbf{j} + (z + x^2y^2)\mathbf{k} \).

Problem 4  Let
\[ F = \frac{-y \mathbf{i} + x \mathbf{j}}{x^2 + y^2}. \]
Note that \( F \) is defined on \( \{(x,y) \in \mathbb{R} \mid (x, y) \neq (0, 0)\} \).
(a) Evaluate \( \int_{C_1} F \cdot d\mathbf{r} \), where \( C_1 \) is the circle \( x^2 + y^2 = 1 \), oriented counterclockwise.
(b) Compute \( \text{curl} \ F \).
(c) Use Green’s Theorem to evaluate \( \int_{C_2} F \cdot d\mathbf{r} \), where \( C_2 \) is the circle \( (x - 2)^2 + (y - 2)^2 = 1 \), oriented counterclockwise.
(d) Is \( F \) conservative?
Problem 5  Let $E$ be a solid in the first octant bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 1$. Evaluate $\iiint_E xyz^2 dV$.

Problem 6  Use the Divergence Theorem to evaluate $\iint_S F \cdot dS$, where

$$F = e^{y^2}i + (y + \sin(z^2))j + (z - 1)k,$$

and $S$ is the upper hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$, oriented upward. Note that the surface $S$ does NOT include the bottom of the hemisphere.

Problem 7  Use Stokes’ Theorem to evaluate $\int_C F \cdot dr$, where

$$F = x^2y^2i - xy^2j + z^3k,$$

and $C$ is the curve of intersection of the plane $3x + 2y + z = 6$ and the cylinder $x^2 + y^2 = 4$, oriented clockwise when viewed from above.

Problem 8  Use Stokes’ Theorem to evaluate $\iint_S \text{curl} F \cdot dS$, where

$$F = (\sin(y + z) - yx^2 - \frac{y^3}{3})i + x \cos(y + z)j + \cos(2y)k,$$

and $S$ consists of the top and the four sides (but not the bottom) of the cube with vertices $(\pm 1, \pm 1, \pm 1)$, oriented outward.

Problem 9  Write in the form of $a + bi$:

(a) Find all the fourth roots of $-4$.

(b) Evaluate $(1 - i)^{10}$.

(c) Find all the possible values of $(-2)^i$.

Problem 10  Let $f(z) = e^{iz}$.

(a) Write $f(z)$ in the form $u + iv$.

(b) Is $f(z)$ analytic?

Problem 11  Let $f(z)$ be an analytic function which only takes real values, i.e., $\text{Im} f(z) = 0$. Show that $f(z)$ is a constant function. (Hint: Use Cauchy-Riemann equations.)