Midterm: MATHS1202 Calculus IV

06/08/23

Name: _____

Instructions — Read this now

- This test has 5 problems on 6 pages (including this page)
- Write your name in the space provided, right now.
- Write you answers in the space provided. If your answers are not written in the space provided, they may not be seen. If you need more space, write on the back side of the preceding sheet, but be sure to label your work clearly.
- Show your work. To receive full credit, your answers must be neatly written and logically organize.
- This is a closed book exam. You can use calculators.
- You are allowed to bring a formula sheet with you with unlimited paper amount. Please hand in your formula sheet along with the exam.
- Academic integrity is expected of all students of Columbia University at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination

_____

Signature of student

	1	2	3	4	5
Score					

1 (10 pts). Let R be the following region

$$R = \{(x, y) \mid 0 \leqslant x \leqslant 5,\, 0 \leqslant y \leqslant \pi\}.$$

Let $f(x, y) = x^3\cos^2(y)$, compute the following integral,

$$\iint_R f(x, y)\mathrm{d}A.$$

2. Consider the following double integral,

$$\int_0^5 \int_0^{\sqrt{25-x^2}} e^{-x^2-y^2} \mathrm{d}y\mathrm{d}x.$$

(a) (5 pts) Sketch the integral region.

(b) (5 pts) Evaluate the integral by changing to polar coordinates.

3 (10 pts). Find the area of the surface $z = \sqrt{x^2 + y^2}$ that lies above the disk $x^2 + y^2 \leqslant 1$.

4 (10 pts). Evaluate the following triple integral by changing to cylindrical coordinates,

$$\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{\sqrt{x^2+y^2}}^{3} yz\mathrm{d}z\mathrm{d}y\mathrm{d}x.$$

5 (10 pts). Let a be a positive real number, evaluate the following triple integral by changing to spherical coordinates,

$$\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}\int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}}\sqrt{x^2+y^2+z^2}\mathrm{d}z\mathrm{d}y\mathrm{d}x.$$