SAMPLE FINAL EXAM FOR MATHS1202

INSTRCUTOR: BAIQING ZHU

1. Compute the following integrals.

(a) (10 pts)

$$\int_{-3}^{0}\int_{-\sqrt{9-x^2}}^{0} e^{-x^2-y^2}\,\mathrm{d}y\mathrm{d}x.$$

(b) (10 pts) Let C be a curve parameterized by $\vec{r}(t) = (2t^2, (1-t)e^t)$, and $\vec{F}(x,y) = (e^y, xe^y + 2y)$ be a vector field, compute

$$\int_C \vec{F}\cdot \mathrm{d}\vec{r}$$

2 (a) (10 pts) Let E be the ellipse $(2x + y)^2 + (x - y)^2 = 9$, consider the transformation $x = u + v$ and $y = u - 2v$, find the equation of the ellipse E in terms of u and v.

(b) (10 pts) Find the integral of the function $f(x, y) = 1$ over the region R bounded by the ellipse E given in (a).

3. (a) (10 pts) State Green's theorem for line integrals.

(b) (10 pts) Use Green's theorem to find the line integral of the field $\vec{F}(x, y) = (\sin(x^3 + 1), e^{\cos(y^2)})$ over the unit circle, oriented counterclockwise.

4. (a) (10 pts) Let $\vec{F}$ be a vector field and C be a smooth curve which is also a boundary of a smooth surface S. State Stokes' theorem.

(b) (10 pts) Use Stokes' theorem to find the following integral: Let $\vec{F}(x, y, z) = (z^3 + y^2, x, z^3 + e^{z^5})$ and C be the intersection of the sphere $x^2 + y^2 + z^2 = 5$ with the plane $z = 1$, oriented counterclockwise as we look from above.

5. (a) (10 pts) Let E be a simple solid region and let S be the boundary surface of E, given with positive (outward) orientation. Let $\vec{F}$ be a vector field whose component functions have continuous partial derivatives on an open region that contains E. State the divergence theorem.

(b) (10 pts) Let E be the simple solid region $[0,1] \times [1,4] \times [0,2]$, and S be the boundary surface of E. Let $\vec{F}(x,y,z) = (4x^2yz, 4xy^2z, 4xyz^2 + \cos(x^4))$. Use the divergence theorem to find the following total flux

$$\iint_S \vec{F} \cdot \mathrm{d}\vec{S}$$