Practice problems for final exam

Final to be held Wednesday, May 13, 1:10pm - 4:00pm

We will have a problem session in preparation for this final:

- Monday, May 11, 8:00pm - 10:00pm, 507 Mathematics

[**1**] Let $f(x, y, z) = x^2y + x^2z + xy^2 + y^2z + xz^2 + yz^2$. Express $f(x)$ as a polynomial $g(\sigma_1, \sigma_2, \sigma_3)$ where σ_1, σ_2, σ_3 are the elementary symmetric functions

$$\sigma_1 = x + y + z, \quad \sigma_2 = xy + xz + yz, \quad \sigma_3 = xyz.$$

[**2**] Let $f(x) = x^2 + ax + b$ have roots α_1 and α_2, where

$$\alpha_1 + \alpha_2 = c, \qquad \alpha_1^2 + \alpha_2^2 = d.$$

Express a and b in terms of c and d.

Recall that the discriminant of $f(x) = x^2 + bx + c$ is $D = b^2 - 4c$, and that the discriminant of $f(x) = x^3 + px + q$ is $D = -4p^3 - 27q^2$.

[**3**] Let $f(x) = x^3 - 2$.

(a) What is the degree of the splitting field K of f over $\mathbb{Q}$?

(b) What is the Galois group $G = G(K/\mathbb{Q})$ of f?

(c) List the subfields L of K, and the corresponding subgroups $H = G(K/L)$ of G.

[**4**] Repeat [**3**] for $f(x) = x^3 - 3x + 1$.

[**5**] Repeat [**3**] for $f(x) = x^4 - 3x^2 + 2$.

[**6**] Repeat [**3**] for $f(x) = x^4 - 5x^2 + 6$.

[**7**] Prove the primitive element theorem (14.4.1, p. 552): Let K be a finite extension of a field F of characteristic zero. There is an element $\gamma \in K$ such that $K = F(\gamma)$.

[**8**] Prove the following theorem about Kummer extensions (14.7.4, p. 566): Let F be a subfield of $\mathbf{C}$ which contains the pth root of unity ζ for a prime p, and let K/F be a Galois extension of degree p. Then K is obtained by adjoining a pth root to F.