Second Midterm Exam

Modern Algebra, Dave Bayer, March 31, 1999

Name: ______________________________

ID: ____________________ **School:** ____________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Each problem is worth 6 points, for a total of 30 points. Please work only one problem per page, and label all continuations in the spaces provided. Extra pages are available. Check your work, where possible.

[**1**] Let G be the abelian group $G = \langle\, a, b, c \mid a^7bc^2 = ab^4c^5 = abc^2 = 1 \,\rangle$. Express G as a product of free and cyclic groups.

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[2] What is the minimal polynomial of $\alpha = \sqrt{-1} + \sqrt{2}$ over $\mathbf{Q}$?

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[3] Consider the module $M = F[x]/(x^3 + 3x^2 + 3x + 1)$ over the ring $R = F[x]$ for a field F.

(a) What is the dimension of M as an F-vector space?

(b) Find a basis for M as an F-vector space, for which the matrix representing multiplication by x is in Jordan canonical form. Give this matrix.

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[4] Prove the *Eisenstein criterion* for irreducibility: Let $f(x) = a_n x^n + \ldots + a_1 x + a_0 \in \mathbb{Z}[x]$, and let p be a prime. If p doesn't divide a_n, p does divide $a_{n-1}, \ldots, a_0$, but p^2 doesn't divide a_0, then $f(x)$ is irreducible as a polynomial in $\mathbf{Q}[x]$.

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[5] Let R be a principal ideal domain, and let

$$I_1 \subset I_2 \subset I_3 \subset \cdots \subset I_n \subset \cdots$$

be an infinite ascending chain of ideals in R. Show that this chain *stabilizes*, i.e.

$$I_N = I_{N+1} = I_{N+2} = \cdots$$

for some N.

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Problem: ______

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