Second midterm

Dave Bayer, Modern Algebra, April 8, 1998

[1] Prove the *Eisenstein criterion* for irreducibility: Let $f(x) = a_n x^n + \ldots + a_1 x + a_0 \in \mathbb{Z}[x]$, and let p be a prime. If p doesn't divide a_n, p does divide $a_{n-1}, \ldots, a_0$, but p^2 doesn't divide a_0, then $f(x)$ is irreducible as a polynomial in $\mathbb{Q}[x]$.

[2] Show that the following polynomials in $\mathbb{Z}[x]$ cannot be factored:
(a) $x^3 + 6x^2 + 9x + 12$
(b) $x^2 + x + 6$

[3] Decide, with proof, whether or not each of the following angles can be constructed.
(a) $\theta = 2\pi/6$
(b) $\theta = 2\pi/7$
(c) $\theta = 2\pi/8$

[4] Let G be the *Abelian* group $G = \langle\, a, b, c \mid b^2c^2 = a^6b^2c^2 = a^6b^4c^4 = 1 \,\rangle$. Express G as a product of free and cyclic groups.

[5] Give a presentation of the finite field with eight elements $\mathbb{F}_8$, of the form $\mathbb{F}_2[x]/(f(x))$. In terms of this presentation, find a generator α of the multiplicative group $\mathbb{F}_8^*$.