db89

0 1 2 3 4 5 6

S16 Final Problem 1
Combinatorics, Dave Bayer

test1a3p1

Test 1

Name Solutions Uni

db89 Dave Bayer

[1] How many ways can one make change for 50 cents, using pennies, nickels, and dimes?

	5	10	15	20	25	30	35	40	45	50
1	1	1	1	1	1	1	1	1	1	1
5	2	3	4	5	6	7	8	9	10	11
10		4		9		16		25		36

Check: Make 0, 10, 20, 30, 40, 50 using 1, 5, finish with dimes

$$\underbrace{1+3+5}_{9}+\underbrace{7+9+11}_{9\ 9\ 9} = 36$$

S16 Final Problem 2
Combinatorics, Dave Bayer

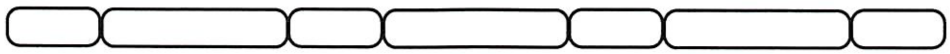
test1a3p2

db89

0 1 2 3 4 5 6

Test 1

[2] How many ways can one fill a tube of length 10, using sticks of length 1 and 2?



$f(10) = f(9) + f(8)$

0	1
1	1
2	2
3	3
4	5
5	8
6	13
7	21
8	34
9	55
10	89

89

db89

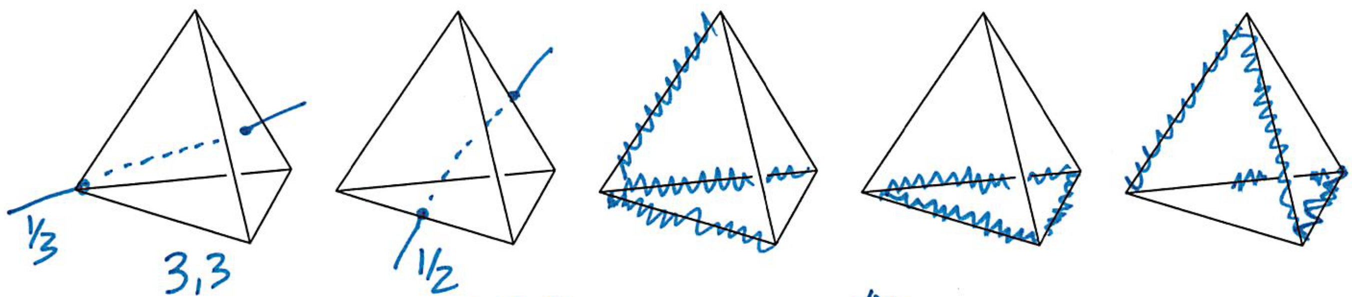
0 1 2 3 4 5 6

S16 Final Problem 3
Combinatorics, Dave Bayer

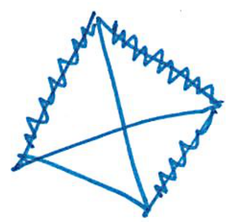
test1a3p3

Test 1

[3] Up to rotational symmetry, how many ways can we choose three of the six edges of a tetrahedron?



$|G| = 12$



Id	1	$\binom{6}{3} = 20$	20
V	8	2	16
e	3	4	12
Σ	12		$48 \div 12 = 4$

db89

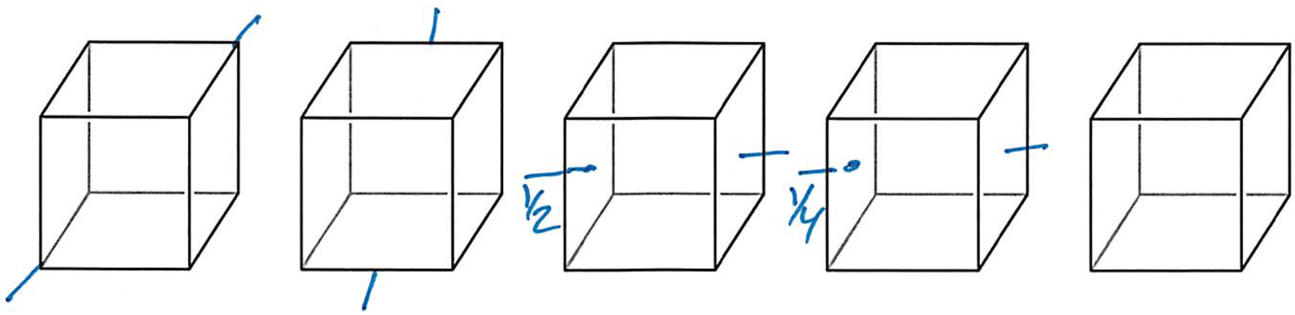
0 1 2 3 4 5 6

S16 Final Problem 4
Combinatorics, Dave Bayer

test1a3p4

Test 1

[4] Up to rotational symmetry, how many ways can we color the six faces of a cube, using at most two colors?



$|G| = 8 \cdot 3 = 24$

Id	1	111111	64	64	96 4
1/3 VV	8	33	4	32	
1/2 ee	6	222	8	48	2
1/4 ff	6	114	8	48	2
1/2 ff	3	1122	16	48	2
	24			÷24	

$\boxed{10}$ ✓

db89

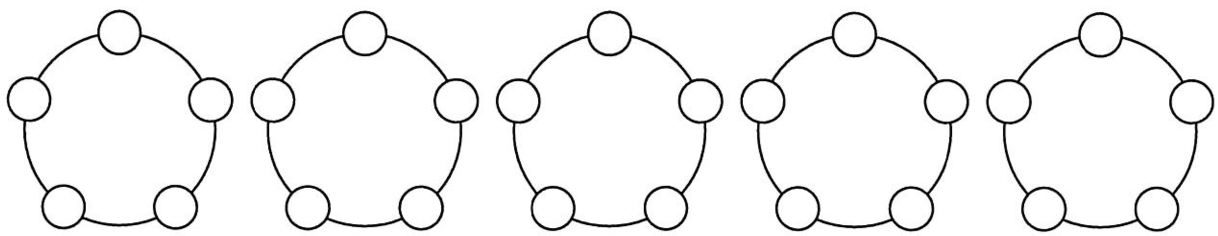
0 1 2 3 4 5 6

S16 Final Problem 5
Combinatorics, Dave Bayer

test1a3p5

Test 1

[5] Up to symmetry, how many ways can the beads of a five bead necklace be colored, using exactly three colors? Consider only rotations, and use all three colors.



Every pattern using all 3 colors has orbit of size 5

raw patterns ÷ 5

$$\begin{aligned} 3^5 &= 243 \\ -3\cdot 2^5 &= -96 \\ +3\cdot 1^5 &= +3 \end{aligned}$$

$246 - 96 = 150 \div 5 = \boxed{30}$

S16 Final Problem 6
Combinatorics, Dave Bayer

test1a3p6

db89

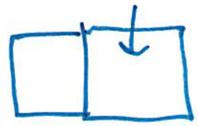
○ ○ ○ ○ ○ ○ ●
0 1 2 3 4 5 6

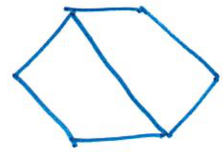
Test 1

[6] For each of the following Young tableaux, find the corresponding polygon dissection using Stanley's correspondence.

1	3	4	6
2	5		

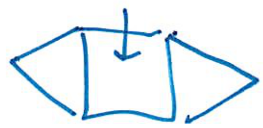
1 2 3 4 5 6
+ + ⊖ − ⊖ −
2 2

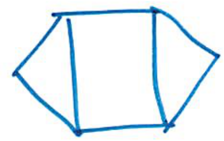




1	3	4
2	5	
6	7	

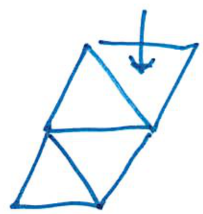
1 2 3 4 5 6 7
+ + ⊖ − ⊖ + ⊖
2 1 1

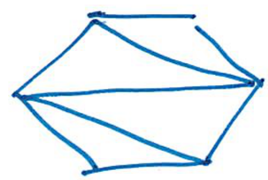




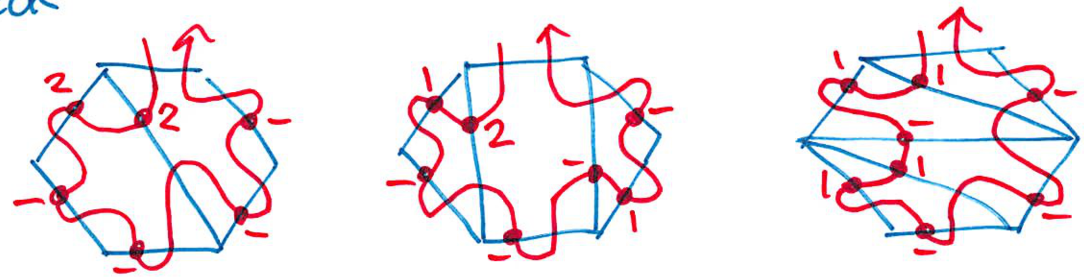
1	3
2	6
4	7
5	8

1 2 3 4 5 6 7 8
+ + ⊖ + + ⊖ ⊖ ⊖





check



1 2 3 4 5 6
2 2 ⊖ − ⊖ −

1 2 3 4 5 6 7
2 1 ⊖ − ⊖ 1 ⊖

1 2 ~~8~~3 4 5 6 7 8
1 1 − 1 1 − − −

1 3 4 6
2 5

1 3 4
2 5
6 7

1 3
2 6
4 7
5 8

S16 Final Problem 7
Combinatorics, Dave Bayer

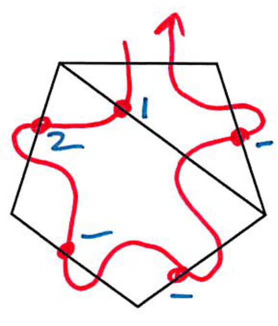
test1a3p7

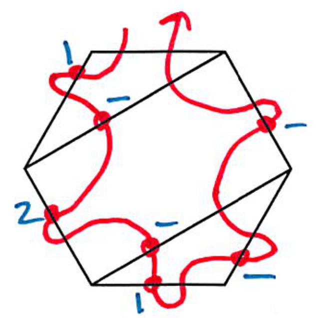
db89

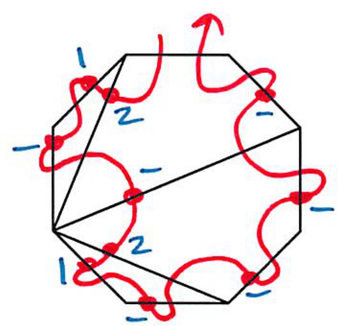
○ ○ ○ ○ ○ ○ ○ ●
0 1 2 3 4 5 6

Test 1

[7] For each of the following polygon dissections, find the corresponding Young tableau using Stanley's correspondence.







1	2	3	4	5
1	2	−	−	−

1	3	5
2	4	

1	2	3	4	5	6	7
1	−	2	−	1	−	−

1	2	6
3	4	
5	7	

1	2	3	4	5	6	7	8	9	10
2	1	−	−	2	1	−	−	−	−

1	3	4	9
2	7		
5	8		
6	10		

✓ ✓ ✓