Homework 2

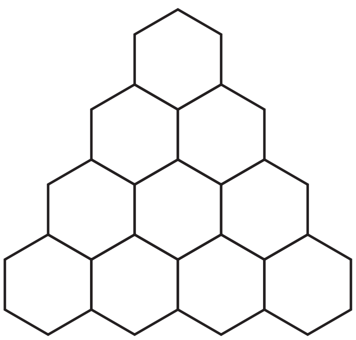
Combinatorics, Dave Bayer, Spring 2015

[1] Give a proof of Burnside's Lemma: If a group G acts on a set of patterns X, then the number of distinct patterns up to symmetry is equal to the average number of patterns fixed by an element of the group:

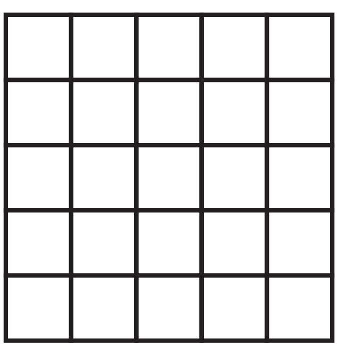
$$\frac{1}{|G|}\sum_{g\in G}|X^g|$$

[2] Up to rotation, how many necklaces have four red beads and four blue beads?

[3] Up to symmetry (rotations and flips), how many ways can one mark two cells of this figure?



[4] Up to symmetry (rotations and flips), how many ways can one mark three squares of this figure?



[5] Up to symmetry, how many ways can the six sides of a cube be colored red, green, or blue?