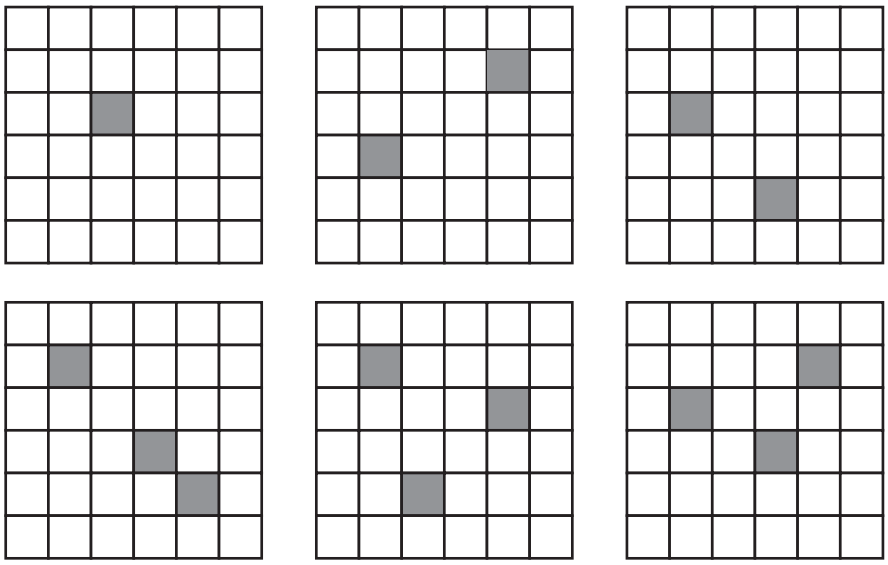
Homework 1

Combinatorics, Dave Bayer, Spring 2015

Work as many of the problems below as interest you. Some do not have clean solutions, but can be counted somehow. If you find yourself wanting to explore a problem and its variations in depth, rather than moving on to other problems, go for it.

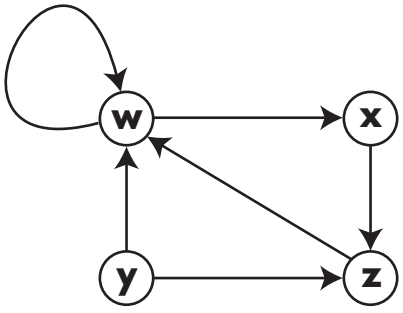
Your work will be scanned, so please use good paper. Ordinary printer paper scans well. Inexpensive "binder" paper, or paper torn from a spiral notebook, can be very difficult to scan.

[1] How many paths are there from the lower left square to the upper right square of each grid below, moving only up or to the right, without passing through any shaded squares?



Leave your answers as expressions involving binomial coefficients; the form of the answer is more interesting than the actual number. As one varies the grids and the positions of the shaded squares, how many types of answers are there? I have shown the only type for one shaded square, and both types for two shaded squares. What about three or four shaded squares?

[2] Without using matrix multiplication, count the number of paths of length ten from w to itself.



[3] Let x and y have degree 1, and let z have degree 3. Count the number of monomials in x, y, and z of degree 12.

[4] Six people are seated around a round table.
(a) How many ways can they be reseated, so everyone moves to a new chair?
(b) How many ways can they be reseated, so everyone has new neighbors on both sides?

[5] There are five ways to fully parenthesize the product $abcd$:

$$a(b(cd)) \qquad a((bc)d) \qquad (ab)(cd) \qquad (a(bc))d \qquad ((ab)c)d$$

How many ways are there to fully parenthesize the product $abcdef$?

[6] How many ways are there to make change for 40 cents, using pennies, nickels, dimes and quarters?

[7] How many ways are there to choose three of the integers $1, 2, 3, 4, 5, 6, 7, 8$ without choosing adjacent integers?

[8] There are three ways to cut a hexagon into two squares. How many ways are there to cut a 12-gon into five squares?