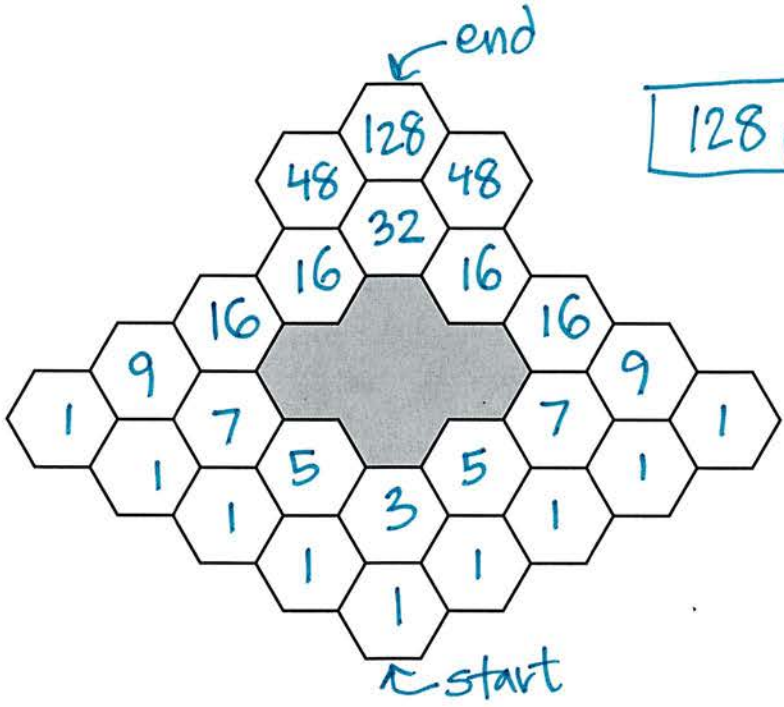
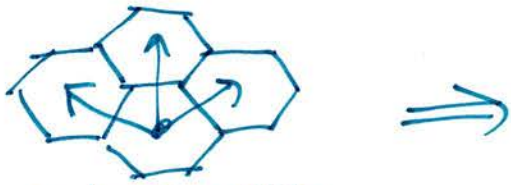
S15 Exam 1 Problem 1
Combinatorics, Dave Bayer

[1] Starting from the bottom hexagon, how many paths are there to the top hexagon, if one moves upward at each step to one of the three (or fewer) neighboring hexagons that is higher?



128 paths



up to 3 moves from each cell, to higher cells that touch

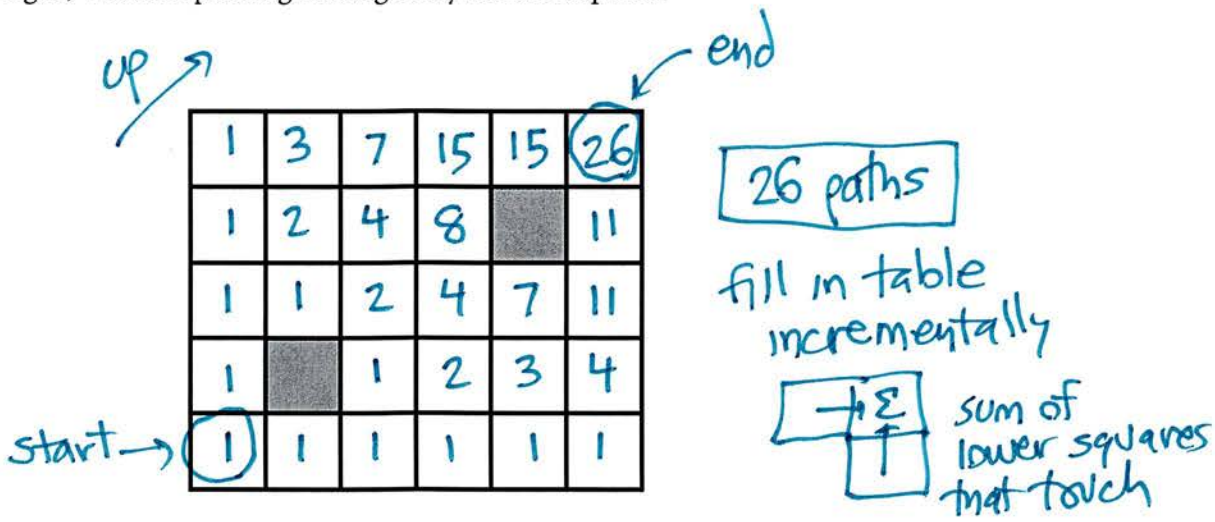
ways to reach each cell is sum of lower cells that touch

Compute incrementally, working up from start

Figure is symmetric, so compute one side and copy for other.

S15 Exam 1 Problem 2
Combinatorics, Dave Bayer

[2] How many paths are there from the lower left square to the upper right square of the grid below, moving only up or to the right, without passing through any shaded square?



check: $\binom{9}{4} - \binom{2}{1}\binom{7}{3} - \binom{7}{3}\binom{2}{1} + \binom{2}{1}\binom{5}{2}\binom{2}{1}$

by inclusion-exclusion on two forbidden squares

$$\frac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2\cdot 1} - 2\cdot 2\cdot \frac{7\cdot 6\cdot 5}{3\cdot 2\cdot 1} + 2\cdot \frac{5\cdot 4}{2\cdot 1}\cdot 2$$

$$9\cdot 14 - 4\cdot 35 + 40$$

$$126 - 140 + 40 = 26 \checkmark$$

S15 Exam 1 Problem 3
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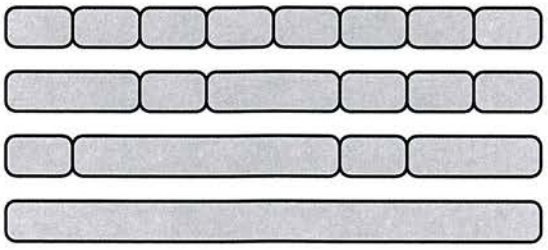
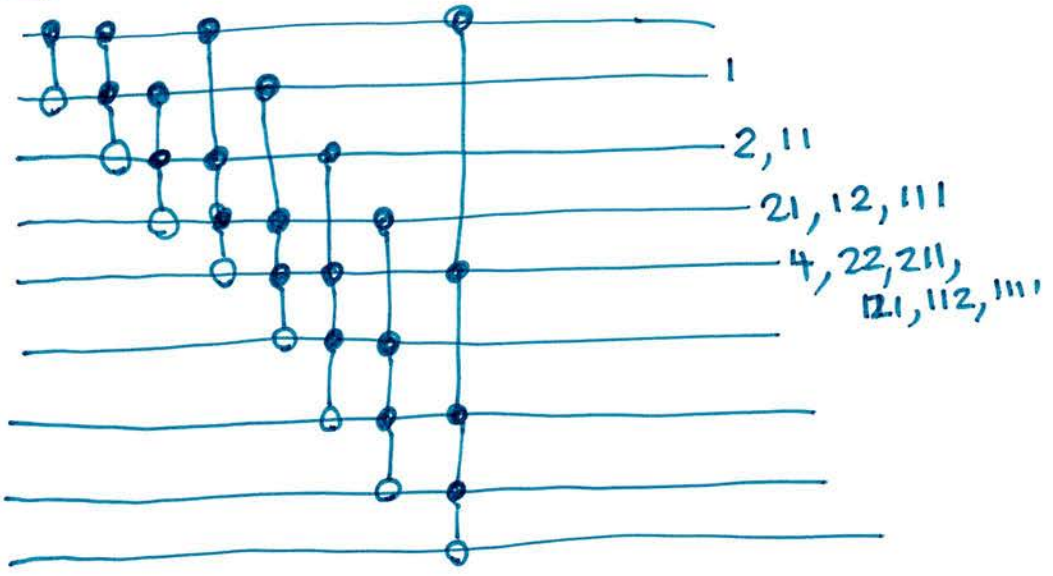
[3] How many ways can one fill a tube of length 8, using sticks of length 1, 2, 4, or 8?

Four of the possibilities are shown below:



length	#ways
0	1
1	1
2	2
3	3
4	6
5	10
6	18
7	31
8	56



56 ways

S15 Exam 1 Problem 4
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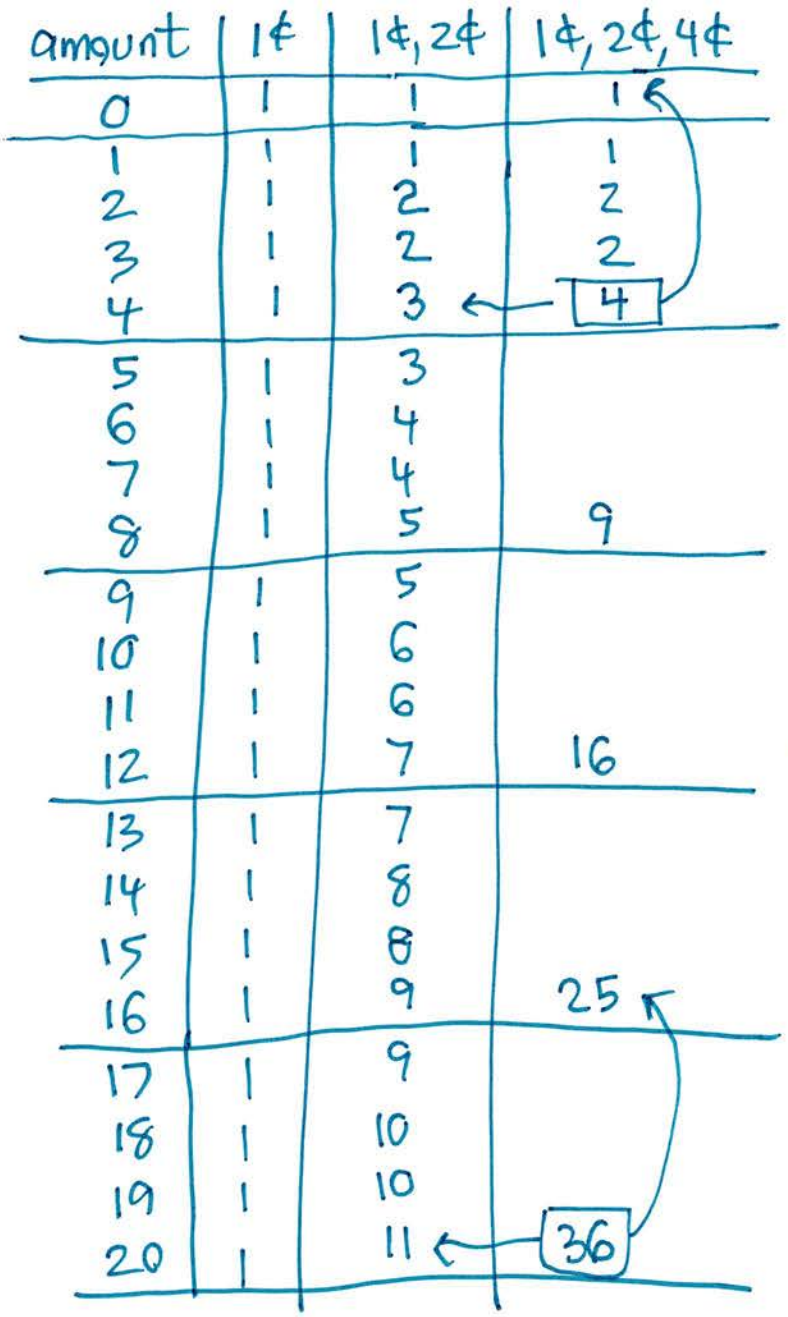
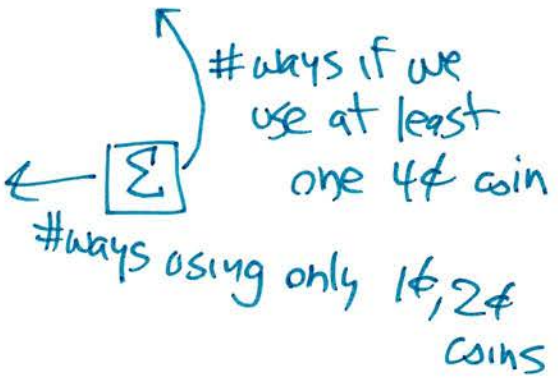
[4] How many ways can one make change for 20 cents, using 1 cent, 2 cent, and 4 cent coins?





only need to consider every 4th entry for this column (and 2nd entry for middle column)

36 ways

S15 Exam 1 Problem 5
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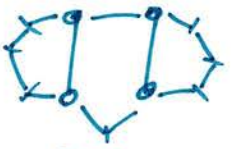
[5] How many ways can one cut an 11-gon into three 5-gons?

Do the cuts meet at a vertex?



each of these cases has 11 rotations, so 11+11 = 22.

22 ways

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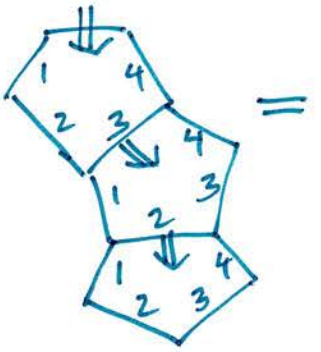
check: Look at tree of cells, starting from top edge.



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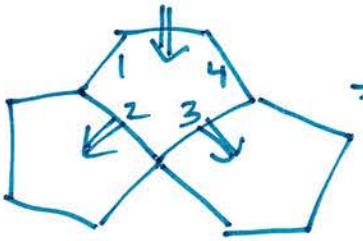
16 ways $\binom{4}{1}\cdot\binom{4}{1}$ $\begin{matrix}1..4\\1..4\end{matrix}$ to label trees for this case



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6 ways to label trees for $\binom{4}{2}$ this case.

16+6 = 22 ✓