Exam 2
Combinatorics, Dave Bayer, March 6, 2014

Solutions

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If you need more that one page for a problem, clearly indicate on each page where to look next for your work.

[1] How many ways can a 12-gon be dissected into two pieces, neither of which is a triangle?

\[
\binom{12}{2} \text{ pairs} - 12 \text{ outside edges} - 12 \text{ triangles}
\]

\[
\frac{12 \cdot 11}{2 \cdot 1} - 2 \cdot 12 = \frac{12(11-4)}{2} = 6.7 = \boxed{42}
\]
[2] How many ways can one draw two crossing interior edges, inside a 12-gon?

Every 4 element subset of vertices yields one cross

\[
\binom{12}{4} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{11 \cdot 5 \cdot 9}{1} = 11 \cdot 45 = 450 + 45 = 495
\]
[3] How many standard Young tableaux are there of the following shape?

\[
\begin{array}{ccc}
5 & 4 & 2 \\
4 & 3 & 1 \\
2 & 1 \\
\end{array}
\]

\[
\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 7! = 42
\]

\[
\frac{n!}{\prod \text{hook lengths}}
\]
[4] Translate each of the following hexagon dissections into standard Young tableaux.
[5] Translate each of the following standard Young tableaux into hexagon dissections.