Practice Final Exam

Modern Algebra I, Dave Bayer, April 26, 2010

The following problems are representative of topics and problems that may appear on our final exam.

[**1**] Let A_5 be the alternating group of even permutations on five elements. Define a *simple* group. Prove that A_5 is simple.

(You may assume that A_5 is isomorphic to the icosahedral group I of rotational symmetries of a dodecahedron, if you prefer working with that group.)

[**2**] (Sylow 1) Let G be a group of order n, and let p be a prime number such that p^e divides n. Prove that there exists a subgroup $P < G$ of order p^e.

[**3**] (Sylow 2) Let G be a group of order $p^e m$, where p be a prime number relatively prime to m. Prove that any two subgroups $P, Q < G$ of order p^e are conjugate.

[**4**] (Sylow 3) Let G be a group of order $p^e m$, where p be a prime number relatively prime to m. Prove that the number of subgroups $P < G$ of order p^e is congruent to 1 mod p.

[**5**] Show that the only group of order 77 is cyclic.

[**6**] There are two groups of order 381. Describe them.

[**7**] There are four groups of order 316. Describe them.

[**8**] There are five groups of order 98. Describe them.

[**9**] Describe the p-Sylow subgroups of the symmetric group S_4, for each prime p dividing $|S_4|$.

[**10**] Describe the p-Sylow subgroups of the alternating group A_5, for each prime p dividing $|A_5|$.

[**11**] Describe the p-Sylow subgroups of the dihedral group D_9, for each prime p dividing $|D_9|$.

[**12**] Describe the p-Sylow subgroups of the dihedral group D_{10}, for each prime p dividing $|D_{10}|$.