Makeup Exam 1

Modern Algebra I, Dave Bayer, February 17, 2009

Name: ______________________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[1] Let C_{10} be the cyclic group of order 10. Using generators and relations, we can write

$$C_{10} \quad = \quad < a \mid a^{10} = 1 >$$

Working instead additively, we can write

$$(\ \mathbb{Z}/10\mathbb{Z}, +\ ) \quad = \quad \{\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9\ \} \quad \text{mod } 10$$

List the subgroups of C_{10}, and draw the lattice of subgroups by inclusion. Which subgroups are normal? What are the corresponding quotient groups? (You may use different notation if you prefer.)

[**2**] Let D_4 be the dihedral group of order 8 (the symmetries of the square). Using generators and relations, we can write

$$D_4 \quad = \quad < a, b \mid a^4 = 1,\ b^2 = 1,\ ba = a^{-1}b >$$

where a is a rotation and b is a flip. List the subgroups of D_4, and draw the lattice of subgroups by inclusion. Which subgroups are normal? What are the corresponding quotient groups? (You may use different notation if you prefer.)

[3] Let S_3 be the symmetric group of all permutations of $\{1,2,3\}$, given in cycle notation as

$$S_3 \quad = \quad \{\, (), \, (1\,2), \, (1\,3), \, (2\,3), \, (1\,2\,3), \, (1\,3\,2) \,\}$$

Let H be the subgroup of S_3 given by

$$H \quad = \quad \{(), \, (1\,2)\}$$

What are the right cosets of H in S_3? What are the left cosets of H in S_3? Is H normal in S_3? If so, what is the quotient group S_3/H? If not, what are the conjugate subgroups gHg^{-1} of H in S_3? (You may use different notation if you prefer.)

[**4**] Let A_4 be the alternating group of even permutations of $\{1,2,3,4\}$, given in cycle notation as

$$\begin{aligned} A_4 \quad = \quad \{ \; & (),\ (1\,2)(3\,4),\ (1\,3)(2\,4),\ (1\,4)(2\,3), \\ & (1\,2\,3),\ (1\,3\,2),\ (1\,2\,4),\ (1\,4\,2),\ (1\,3\,4),\ (1\,4\,3),\ (2\,3\,4),\ (2\,4\,3) \; \} \end{aligned}$$

Let H be the subgroup of A_4 given by

$$H \quad = \quad \{\, (),\ (1\,2)(3\,4),\ (1\,3)(2\,4),\ (1\,4)(2\,3) \,\}$$

What are the right cosets of H in A_4? What are the left cosets of H in A_4? Is H normal in A_4? If so, what is the quotient group A_4/H? If not, what are the conjugate subgroups gHg^{-1} of H in A_4? (You may use different notation if you prefer.)

[5] Let C_4 be the cyclic group of order 4, and let C_6 be the cyclic group of order 6. Using generators and relations, we can write

$$\begin{aligned} C_4 &= < a \mid a^4 = 1 > \\ C_6 &= < b \mid b^6 = 1 > \end{aligned}$$

Working instead additively, we can write

$$\begin{aligned} (\,\mathbb{Z}/4\mathbb{Z}, +\,) &= \{\,0,\ 1,\ 2,\ 3\,\} \quad \text{mod } 4 \\ (\,\mathbb{Z}/6\mathbb{Z}, +\,) &= \{\,0,\ 1,\ 2,\ 3,\ 4,\ 5\,\} \quad \text{mod } 6 \end{aligned}$$

How many group homomorphisms can you find from C_4 to C_6? How many group homomorphisms can you find from C_6 to C_4? (You may use different notation if you prefer.)

[6] Let D_4 be the dihedral group of order 8 (the symmetries of the square), and let Q be the quaternions $\{\pm 1, \pm i, \pm j, \pm k\}$. Using generators and relations, we can write

$$\begin{aligned} D_4 &= \; < a, b \mid a^4 = 1,\ b^2 = 1,\ ba = a^{-1}b > \\ Q &= \; < i, j \mid i^4 = 1,\ j^4 = 1,\ i^2 = j^2,\ ji = i^{-1}j > \end{aligned}$$

where a is a rotation and b is a flip. (Check that these relations on i and j are equivalent to the usual description of the quaternions.) How many group homomorphisms can you find from D_4 to Q? How many group homomorphisms can you find from Q to D_4? (You may use different notation if you prefer.)