
Exam 1

Modern Algebra I, Dave Bayer, February 17, 2009

Name: ____________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[1] Let C_6 be the cyclic group of order 6. Using generators and relations, we can write

$$C_6 \quad = \quad < \; a \mid a^6 = 1 \; >$$

Working instead additively, we can write

$$(\; \mathbb{Z}/6\mathbb{Z}, + \;) \quad = \quad \{ \; 0, \; 1, \; 2, \; 3, \; 4, \; 5 \; \} \quad \text{mod } 6$$

List the subgroups of C_6, and draw the lattice of subgroups by inclusion. Which subgroups are normal? What are the corresponding quotient groups? (You may use different notation if you prefer.)

[2] Let D_3 be the dihedral group of order 6 (the symmetries of the triangle). Using generators and relations, we can write

$$D_3 \quad = \quad < a, b \mid a^3 = 1,\ b^2 = 1,\ ba = a^{-1}b >$$

where a is a rotation and b is a flip. List the subgroups of D_3, and draw the lattice of subgroups by inclusion. Which subgroups are normal? What are the corresponding quotient groups? (You may use different notation if you prefer.)

[3] Let S_3 be the symmetric group of all permutations of $\{1,2,3\}$, given in cycle notation as

$$S_3 \quad = \quad \{\, (), \ (12), \ (13), \ (23), \ (123), \ (132) \,\}$$

Let H be the subgroup of S_3 given by

$$H \quad = \quad \{\, (), \ (123), \ (132) \,\}$$

What are the right cosets of H in S_3? What are the left cosets of H in S_3? Is H normal in S_3? If so, what is the quotient group S_3/H? If not, what are the conjugate subgroups gHg^{-1} of H in S_3? (You may use different notation if you prefer.)

[4] Let A_4 be the alternating group of even permutations of $\{1,2,3,4\}$, given in cycle notation as

$$\begin{aligned} A_4 \quad = \quad \{ \quad & (), (12)(34), (13)(24), (14)(23), \\ & (123), (132), (124), (142), (134), (143), (234), (243) \quad \} \end{aligned}$$

Let H be the subgroup of A_4 given by

$$H \quad = \quad \{(), (123), (132)\}$$

What are the right cosets of H in A_4? What are the left cosets of H in A_4? Is H normal in A_4? If so, what is the quotient group A_4/H? If not, what are the conjugate subgroups gHg^{-1} of H in A_4? (You may use different notation if you prefer.)

[5] Let C_6 be the cyclic group of order 6, and let C_9 be the cyclic group of order 9. Using generators and relations, we can write

$$\begin{aligned} C_6 &= < a \mid a^6 = 1 > \\ C_9 &= < b \mid b^9 = 1 > \end{aligned}$$

Working instead additively, we can write

$$\begin{aligned} (\mathbb{Z}/6\mathbb{Z}, +) &= \{0, 1, 2, 3, 4, 5\} \quad \text{mod } 6 \\ (\mathbb{Z}/9\mathbb{Z}, +) &= \{0, 1, 2, 3, 4, 5, 6, 7, 8\} \quad \text{mod } 9 \end{aligned}$$

How many group homomorphisms can you find from C_6 to C_9? How many group homomorphisms can you find from C_9 to C_6? (You may use different notation if you prefer.)

[**6**] Let $C_2 \times C_2$ be the Klein-4 group of order 4 (a product of two cyclic groups of order 2), and let D_4 be the dihedral group of order 8 (the symmetries of the square). Using generators and relations, we can write

$$\begin{aligned} C_2 \times C_2 &= \; < a, b \mid a^2 = 1,\ b^2 = 1,\ ba = ab > \\ D_4 &= \; < c, d \mid c^4 = 1,\ d^2 = 1,\ dc = c^{-1}d > \end{aligned}$$

where c is a rotation and d is a flip. How many group homomorphisms can you find from $C_2 \times C_2$ to D_4? How many group homomorphisms can you find from D_4 to $C_2 \times C_2$? (You may use different notation if you prefer.)