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## Practice Exam 2

Modern Algebra I, Dave Bayer, April 1, 2008

Name: \_\_\_\_\_

[1] (6 pts)	[2] (6 pts)	[3] (6 pts)	[4] (6 pts)	[5] (6 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[1] How many different necklaces can be made from 6 red or blue beads, if we consider rotations to be the same necklace?

[2] How many ways can 3 checkers be placed on a 4 by 4 checkerboard, if two arrangements are considered the same if they differ by a symmetry of the dihedral group  $D_4$ ?

[3] The *Klein four* group  $V$  is the group of order 4 with elements

$$\{1, a, b, c\}$$

and the multiplication rules

$$a * a = b * b = c * c = 1, \quad a * b = b * a = c, \quad b * c = c * b = a, \quad c * a = a * c = b$$

1. Find two groups of order 10 which have the cyclic group  $C_5$  of order 5 as a normal subgroup.
2. Find two groups of order 12 which have the Klein-4 group  $V$  of order 4 as a normal subgroup.

[4] The *Quaternion* group  $Q$  is the group of order 8 with elements

$$\{1, -1, i, -i, j, -j, k, -k\}$$

and the multiplication rules

$$i * i = j * j = k * k = -1, \quad i * j = -j * i = k, \quad j * k = -k * j = i, \quad k * i = -i * k = j$$

Find a nontrivial normal subgroup  $N$  of  $Q$ . For your choice of  $N$ , what is the quotient group  $Q/N$ ?

[5] Let  $Q$  be the Quaternion group of order 8, and let  $H$  be the cyclic subgroup of order 4 generated by the element  $i$ .

1. Is  $H$  a normal subgroup of  $Q$ ? Why or why not?
2. Let  $X$  be the set of all 4-element subsets of  $Q$ . Let  $Q$  act on  $X$  by conjugation.  $H$  is an element of  $X$ ; what is the size of its orbit?
3. How many orbits are there, for this action of  $Q$  on  $X$ ?