Exam 2

Modern Algebra I, Dave Bayer, April 1, 2008

Name: _____

[1] (6 pts)	[2] (6 pts)	[3] (6 pts)	[4] (6 pts)	[5] (6 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[1] How many different necklaces can be made from 12 red or blue beads, if we consider rotations to be the same necklace?

[2] How many ways can 4 checkers be placed on a 4 by 4 checkerboard, if two arrangements are considered the same if they differ by a symmetry of the dihedral group D_4?

[3]

1. Find two groups of order 8 which have the cyclic group C_4 of order 4 as a normal subgroup.

2. Find two groups of order 18 which have the symmetric group S_3 of order 6 as a normal subgroup.

[**4**] Let A_4 be the alternating group of order 12, of all even permutations of $\{1, 2, 3, 4\}$. Find a nontrivial normal subgroup N of A_4. For your choice of N, what is the quotient group A_4/N?

[5] Let A_4 be the alternating group of order 12, of all even permutations of $\{1, 2, 3, 4\}$. Let H be the cyclic subgroup of order 3 generated by the permutation $(1\,2\,3)$.

1. Is H a normal subgroup of A_4? Why or why not?

2. Let X be the set of all 3-element subsets of A_4. Let A_4 act on X by conjugation. H is an element of X; what is the size of its orbit?

3. How many orbits are there, for this action of A_4 on X?