Exam 1

Modern Algebra I, Dave Bayer, February 19, 2008

Name: ______________________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[1] Complete each of the following multiplication tables, so that the resulting table is the multiplication rule for a group G. In each case, what group do you get?

$*$	1	2	3	4
1	1	2	3	4
2	2	1		
3	3		1	
4	4			

$*$	1	2	3	4
1	1	2	3	4
2	2	1		
3	3			
4	4		1	

$*$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	3	1	5		4
3	3	1	2	6		5
4	4	6	5			2
5	5					3
6	6	5	4	3	2	

[2] For each of the groups found in problem 1, let the group act on itself by right multiplication. Use these actions to find permutation representations for each group.

[3] Using the given generators, draw the Cayley graphs for each of the following groups.

1. $G = \langle \{1, i, -1, -i\}, \ * \rangle \subset \langle \mathbb{C}, \ * \rangle$, using the generator i.
2. $G = \langle \mathbb{Z}/6\mathbb{Z}, \ + \rangle$, using the generators 2 and 3.
3. $G = S_3$, using the generators $(1\ 2\ 3)$ and $(1\ 2)$.

[4] Here is a multiplication table for the dihedral group D_5 of order 10:

$*$	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	1	7	8	9	10	6
3	3	4	5	1	2	8	9	10	6	7
4	4	5	1	2	3	9	10	6	7	8
5	5	1	2	3	4	10	6	7	8	9
6	6	10	9	8	7	1	5	4	3	2
7	7	6	10	9	8	2	1	5	4	3
8	8	7	6	10	9	3	2	1	5	4
9	9	8	7	6	10	4	3	2	1	5
10	10	9	8	7	6	5	4	3	2	1

1. Find six nontrivial subgroups H of D_5 (not order 1 or 10).
2. For two of the H that you found, what are the right cosets of H in G?

[5] Let G be the group $\langle \mathbb{Z}/3\mathbb{Z},\ + \rangle$, and let H be the group $\langle \mathbb{Z}/6\mathbb{Z},\ + \rangle$. How many group homomorphisms can you find from G to H? From H to G?

[6] The *Klein four* group V is the group of order 4 with elements

$$\{1, a, b, c\}$$

and the multiplication rules

$$a * a = b * b = c * c = 1, \quad a * b = b * a = c, \quad b * c = c * b = a, \quad c * a = a * c = b$$

The *Quaternion* group Q is the group of order 8 with elements

$$\{1, -1, i, -i, j, -j, k, -k\}$$

and the multiplication rules

$$i * i = j * j = k * k = -1, \quad i * j = -j * i = k, \quad j * k = -k * j = i, \quad k * i = -i * k = j$$

How many group homomorphisms can you find from V to Q? From Q to V?