

Rules:



$$AB = 1$$

(Americans approaching intersection on overpass veer off to right.
Brits veer off to left. This convention is opposite that in the literature.)



extra loops $\rightarrow C$

$$C = -A^2 - B^2$$

represented graphically as

$$\frac{B^2}{-} \begin{array}{c} + \\ \ominus \end{array} \frac{A^2}{-}$$

For trivial knot, Kauffman = writhe.

To compute writhe, pick orientation,



$L R$ cancel

$A B$ cancel

$$R R L = R = \boxed{-A^3}$$



$$L = -B^3$$

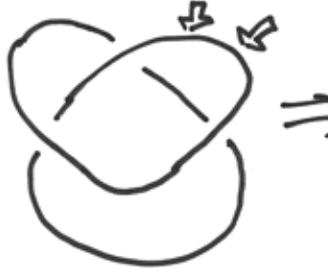


$$R = -A^3$$

In general, can slide knot around like a belt between two sheets of glass,
 II & III Reidemeister moves OK:



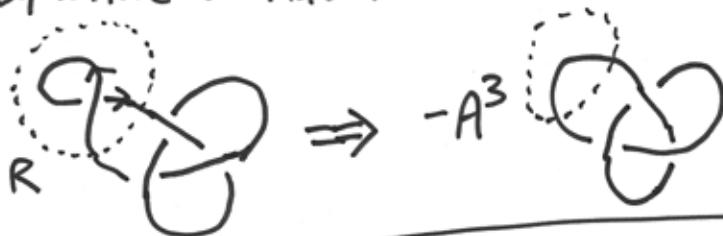
Above example easier this way:



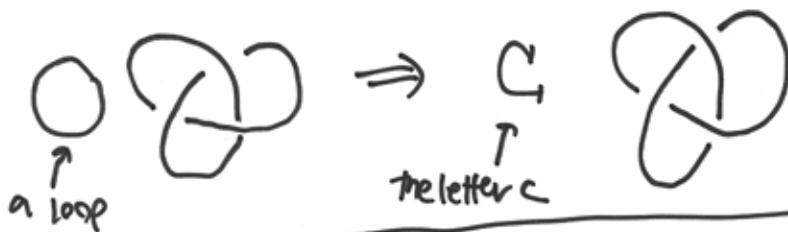
$$R = \boxed{-A^3}$$

push loop over to simplify knot

Removing a twist (Reidemeister I) from any knot multiplies Kauffman by weight of that twist:



Removing a loop from any knot multiplies Kauffman by C:



Nice to remember answers when we get down to something familiar:

$$\text{Diagram of a knot} = -A^4 - B^4$$

work:
 $\text{Diagram of a knot}^* = A + \underbrace{\text{Diagram of a knot}}_{-A^3} + B \text{ Diagram of a knot}$

$$\text{Diagram of a knot} = 1$$

(by convention)
 (don't count 1st loop)

$$\text{Diagram of a knot} = A^7 - A^3 - B^5$$

$$\text{Diagram of a knot} = B^7 - B^3 - A^5$$

(mirror image)

work:

$$\text{Diagram of a knot}^* = A + \underbrace{\text{Diagram of a knot}}_{A^6} + B \text{ Diagram of a knot}$$

$$= A^7 - A^3 - B^5$$

$$\underbrace{\text{Diagram of a knot}}_{-A^4 - B^4}$$



$$= -B^{10} + B^6 - B^2 - A^6$$



$$= -A^{10} + A^6 - A^2 - B^6$$

work:

$$\text{Diagram}^* = A \underbrace{\text{Diagram}}_{B^7 - B^3 - A^5} + B \underbrace{\text{Diagram}}_{-B^9} = -B^{10} + B^6 - B^2 - A^6$$

mirror swaps
A, B

$$\text{Diagram}^* = A \underbrace{\text{Diagram}}_{-A^9} + B \underbrace{\text{Diagram}}_{A^7 - A^3 - B^5} = -A^{10} + A^6 - A^2 - B^6$$

(mirror image)

Sum of two knots or links is product of Kauffman polys:

$$\text{Diagram} = (-A^4 - B^4)(-A^4 - B^4) = A^8 + 1 + 1 + B^8$$

$\boxed{= A^8 + 2 + B^8}$

$(-A^4 - B^4) \cdot (-A^4 - B^4)$

check this example:

$$\text{Diagram}^* = A \underbrace{\text{Diagram}}_R + B \underbrace{\text{Diagram}}_L$$

$$= A(-A^3)(-A^4 - B^4) + B(-B^3)(-A^4 - B^4)$$

$$= A^8 + 1 + 1 + B^8 \quad \square$$

Compute the Kauffman polynomial for each of the following knots and links:

01 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$\Rightarrow C \Rightarrow C = \boxed{1} \text{ unknot}$

02 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$\Rightarrow C \Rightarrow C = \boxed{1} \text{ unknot}$

again

03 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$\Rightarrow C = A \left(\begin{array}{c} R \\ R \\ R \\ R \\ -A^3(-A^4-A^4) \end{array} \right) + B \left(\begin{array}{c} R \\ R \\ R \\ R \\ B^7-B^3-A^5 \end{array} \right) = \boxed{B^8-B^4+1-A^4+A^8}$

figure-8 knot

04 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$\Rightarrow C = -A^3 \left(\begin{array}{c} R \\ R \\ R \\ R \\ -A^3(B^7-B^3-A^5) \end{array} \right) = \boxed{-B^4+1+A^8}$

Compute the Kauffman polynomial for each of the following knots and links:

05

B^{14}	B^{12}	B^{10}	B^8	B^6	B^4	B^2	1	A^2	A^4	A^6	A^8	A^{10}	A^{12}	A^{14}
+						-								

$$= -B^3$$

$$= -B^3(A^7 - A^3 - B^5)$$

$$= [-A^4 + 1 + B^8]$$

06

B^{14}	B^{12}	B^{10}	B^8	B^6	B^4	B^2	1	A^2	A^4	A^6	A^8	A^{10}	A^{12}	A^{14}
+						+								

$$= A$$

$$+ B$$

$$= B^8 + 1 - A^4$$

$$- B^3(-B^4 - A^4)$$

$$-B^3(A^3 - A^2 - B^5)$$

$\text{II (or) } \text{check}$

$$= -A^3$$

(trefoil with 9 twist)

07

B^{14}	B^{12}	B^{10}	B^8	B^6	B^4	B^2	1	A^2	A^4	A^6	A^8	A^{10}	A^{12}	A^{14}
+														

$$= \text{CS} = \text{S} = 1$$

08

B^{14}	B^{12}	B^{10}	B^8	B^6	B^4	B^2	1	A^2	A^4	A^6	A^8	A^{10}	A^{12}	A^{14}
+						-								
+						-								
+						-								

$$= A$$

$$+ B$$

$$= B^8 - B^4 + 2 - A^4 + A^8 - A^{12}$$

$$+ A^6(B^7 - B^3 - A^5)$$

\cong twists trefoil

$$= -A^3$$

$$(-B^{10} + B^6 - B^2 - A^6)$$

Compute the Kauffman polynomial for each of the following knots and links:

09

$$B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$$
 $=$
 $=$
 $=$
 $=$
 $\boxed{1}$

10

$$B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$$
 $=$
 $=$
 A
 $+ B$
 $\underbrace{-A^3(-A^4-B^4)}$
 $\underbrace{B^7-B^3-A^5}$

(looks like figure-8)

 $= \boxed{B^8-B^4+1-A^4+A^8}$

11

$$B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$$
 $=$
 $=$
 $\boxed{B^8-B^4+1-A^4+A^8}$

figure 8 again

mirror image of 10

12

$$B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$$
 $=$
 $=$
 $=$
 $\boxed{1}$

Compute the Kauffman polynomial for each of the following knots and links:

13

$$B^{14} \left[\begin{matrix} - & + & - & + \\ B^{12} & B^{10} & B^8 & B^6 & B^4 & B^2 & 1 & A^2 & A^4 & A^6 & A^8 \end{matrix} \right] A^{10} A^{12} A^{14}$$
 $= A \text{ } \begin{matrix} \text{Diagram of a knot with a crossing labeled R} \\ \text{Diagram of a knot with a crossing labeled L} \end{matrix} + B \text{ } \begin{matrix} \text{Diagram of a knot with a crossing labeled R} \\ \text{Diagram of a knot with a crossing labeled L} \end{matrix} \Rightarrow \text{Diagram of a knot with a crossing labeled L}$
 $-A^3(-A^4-B^4)$
 $\boxed{-B^{12}+B^8-B^4+2-A^4+A^8} = -B^3$

(mirror image of 08)

figure-8
 $B^8-B^4+1-A^4+A^8$

14

$$B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 \left[\begin{matrix} + \\ 1 \end{matrix} \right] A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}$$
 $= \text{Diagram of a knot with a crossing labeled R} = \text{Diagram of a knot with a crossing labeled L} = \boxed{L}$

15

$$B^{14} B^{12} B^{10} B^8 B^6 \left[\begin{matrix} - & + & + \\ B^4 & B^2 & 1 & A^2 & A^4 & A^6 & A^8 \end{matrix} \right] A^{10} A^{12} A^{14}$$
 $= \text{Diagram of a knot with a crossing labeled R} = A \text{ } \begin{matrix} \text{Diagram of a knot with a crossing labeled R} \\ \text{Diagram of a knot with a crossing labeled L} \end{matrix} + B \text{ } \begin{matrix} \text{Diagram of a knot with a crossing labeled R} \\ \text{Diagram of a knot with a crossing labeled L} \end{matrix}$
 $-A^3(-A^4-B^4)$
 $\boxed{-B^4+1+A^8} = -B^3$

(trefoil w/a twist)

16

$$B^{14} B^{12} B^{10} B^8 B^6 \left[\begin{matrix} - & + & + \\ B^4 & B^2 & 1 & A^2 & A^4 & A^6 & A^8 \end{matrix} \right] A^{10} A^{12} A^{14}$$
 $= \text{Diagram of a knot with a crossing labeled R} = -A^3 \text{ } \begin{matrix} \text{Diagram of a knot with a crossing labeled R} \\ \text{Diagram of a knot with a crossing labeled R} \end{matrix} = \boxed{-B^4+1+A^8}$

$B^7-B^3-A^5$

again, trefoil
w/a twist

Compute the Kauffman polynomial for each of the following knots and links:

17 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$$= \text{another twisted trefoil} = -B^3(A^7 - A^3 - B^5) = [+B^8 + 1 - A^4]$$

18 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$$= \text{looks like figure-8} = A * + B \text{ trefoil}$$

$$-A^3(-A^4 - B^4) \quad (B^7 - B^3 - A^5)$$

$$= [B^8 - B^4 + 1 - A^4 + A^8]$$

19 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$$= \text{trefoil} = [1]$$

20 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$$= \text{trefoil} = [1]$$

Compute the Kauffman polynomial for each of the following knots and links:

21 $B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad B^6 \quad B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \quad A^6 \quad A^8 \quad A^{10} \quad A^{12} \quad A^{14}$

$=$ $=$ $\boxed{1}$

22 $B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad B^6 \quad B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \quad A^6 \quad A^8 \quad A^{10} \quad A^{12} \quad A^{14}$

$=$ $=$ $\boxed{1}$

23 $B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad B^6 \quad B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \quad A^6 \quad A^8 \quad A^{10} \quad A^{12} \quad A^{14}$

$=$ $= -B^3$ $= \boxed{B^8 + 1 - A^4}$

$A^7 - A^3 - B^5$

twisted trefoil

24 $B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad B^6 \quad B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \quad A^6 \quad A^8 \quad A^{10} \quad A^{12} \quad A^{14}$

$=$ $=$ $\boxed{2}$

Compute the Kauffman polynomial for each of the following knots and links:

25

$$B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad B^6 \quad \boxed{B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \quad A^6 \quad A^8} \quad A^{10} \quad A^{12} \quad A^{14}$$

$$= -A^3(B^7 - B^3 - A^5)$$

$$= \boxed{-B^4 + 1 + A^8}$$

26

$$B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad B^6 \quad \boxed{-B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \quad A^6 \quad A^8 \quad A^{10} \quad -A^{12}} \quad A^{14}$$

$$* = A \quad \text{Diagram} + B \quad \text{Diagram} = \boxed{-B^{12} + B^8 - B^4 + 3 - A^4 + A^8 - A^{12}}$$

$$= (B^7 - B^3 - A^5)(A^7 - A^3 - B^5)$$

$$A \quad \text{Diagram} + B \quad \text{Diagram}$$

27

$$B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad B^6 \quad \boxed{B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \quad A^6 \quad A^8} \quad A^{10} \quad A^{12} \quad A^{14}$$

$$= -A^3(B^7 - B^3 - A^5)$$

$$= \boxed{-B^4 + 1 + A^8}$$

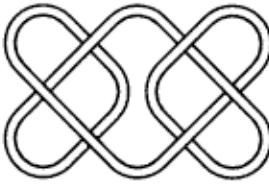
28

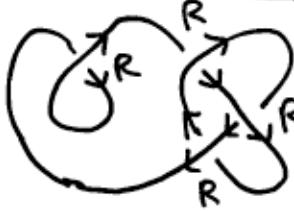
$$B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad B^6 \quad B^4 \quad B^2 \quad \boxed{1} \quad A^2 \quad A^4 \quad A^6 \quad A^8 \quad A^{10} \quad A^{12} \quad A^{14}$$

$$= \boxed{1}$$

Compute the Kauffman polynomial for each of the following knots and links:

29 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \boxed{B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8} \ A^{10} \ A^{12} \ A^{14}$

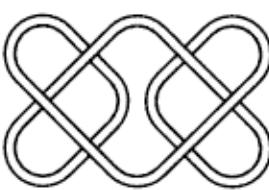


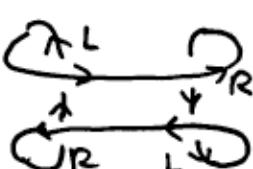
$=$ 

$= -A^3(B^7 - B^3 - A^5)$

$= \boxed{-B^4 + 1 + A^8}$

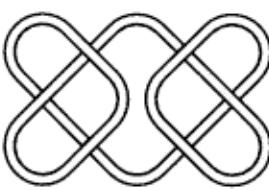
30 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$



$=$ 

$= \boxed{1}$

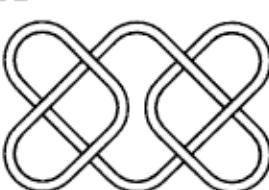
31 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

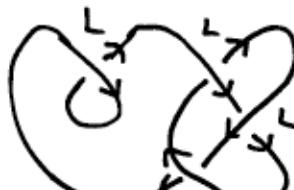


$=$ 

$= \boxed{1}$

32 $B^{14} \ B^{12} \ B^{10} \ \boxed{+ \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ - \ A^4} \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$



$=$ 

$= -B^3(A^7 - A^3 - B^5)$

$= \boxed{B^8 + 1 - A^4}$

Compute the Kauffman polynomial for each of the following knots and links:

33 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$=$ $=$ 1

34 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$=$ $= -B^3(A^7-A^3-B^5)$

$=$ B^8+1-A^4

35 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$=$ $= (A^7-A^3-B^5) \cdot (B^2-B^3-A^5)$

$=$ $-B^{12}+B^8-B^4+3-A^4+A^8-A^{12}$

36 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$=$ $= -B^3(A^7-A^3-B^5)$

$=$ B^8+1-A^4

Compute the Kauffman polynomial for each of the following knots and links:

37 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$=$ $=$ $\boxed{1}$

38 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$=$ $=$ $\boxed{-A^3(B^7-B^3+A^5)}$

$=$ $=$ $\boxed{A^8+1-B^4}$

39 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

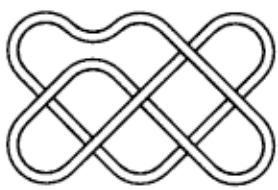
$=$ $=$ $\boxed{1}$

40 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$=$ $=$ $\boxed{1}$

Compute the Kauffman polynomial for each of the following knots and links:

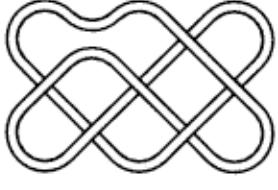
41



$$= \begin{array}{c} B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad B^6 \\ + \end{array} B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \quad A^6 \quad A^8 \quad A^{10} \quad A^{12} \quad A^{14}$$


$$= \boxed{B^6}$$

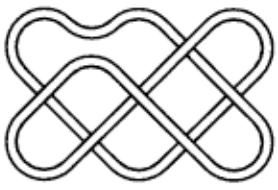
42



$$= \begin{array}{c} B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad B^6 \quad B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \\ + \end{array} A^6 \quad A^8 \quad A^{10} \quad A^{12} \quad A^{14}$$


$$= \boxed{A^6}$$

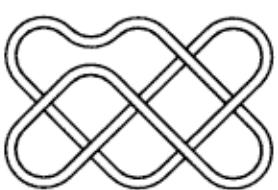
43

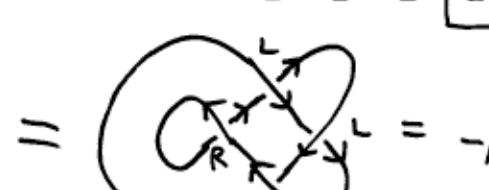


$$= \begin{array}{c} B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad B^6 \quad B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \\ + \end{array} A^6 \quad A^8 \quad A^{10} \quad A^{12} \quad A^{14}$$


$$= \boxed{A^6}$$

44



$$= \begin{array}{c} B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad B^6 \quad B^4 \\ + \end{array} B^2 \quad 1 \quad A^2 \quad A^4 \quad A^6 \quad A^8 \quad - \quad A^{10} \quad A^{12} \quad A^{14}$$


$$= -A^3(A^7 - A^3 - B^5)$$

$$= \boxed{-A^{10} + A^6 + B^2}$$

Compute the Kauffman polynomial for each of the following knots and links:

45 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \boxed{A^6} \ A^8 \ A^{10} \ A^{12} \ A^{14}$

46 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \boxed{A^6} \ A^8 \ A^{10} \ A^{12} \ A^{14}$

47 $B^{14} \ B^{12} \boxed{+} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$A^6(A^2-A^3-B^5)$

$B^{10}-B^6+B^2-2B^4-2A^2+2A^6-2A^{10}+A^{14}$

$(-A^3)(A^2-A^3+B^5)$

$B^8-B^4+1-A^4+A^8$

figure 8

48 $B^{14} \ B^{12} \boxed{-} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$

$A^2(A^8+2+B^8)$

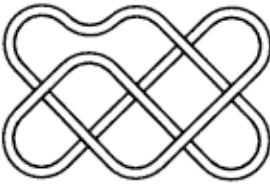
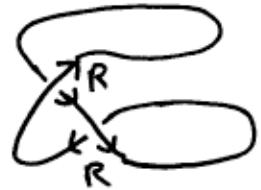
$-A^2-B^2-B^10$

$(-A^4-B^4)-(-A^4-B^4)$

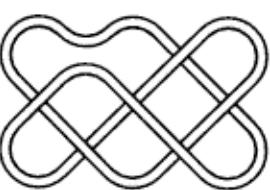
$-A^2-B^2$

Compute the Kauffman polynomial for each of the following knots and links:

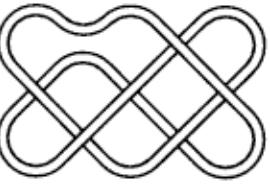
49 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \left(\begin{matrix} A^6 \\ + \end{matrix} \right) A^8 \ A^{10} \ A^{12} \ A^{14}$


 $=$ 
 $=$ $\boxed{A^6}$

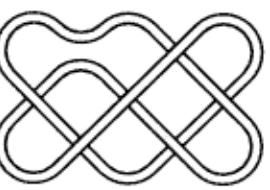
50 $B^{14} \ B^{12} \ B^{10} \ B^8 \ B^6 \ B^4 \ B^2 \ 1 \ A^2 \ A^4 \left(\begin{matrix} A^6 \\ + \end{matrix} \right) A^8 \ A^{10} \ A^{12} \ A^{14}$


 $=$ 
 $=$ $\boxed{A^6}$

51 $B^{14} \ B^{12} \ B^{10} \ B^8 \left(\begin{matrix} + \\ B^6 \end{matrix} \right) B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$


 $=$ 
 $=$ $\boxed{B^6}$
(turbulence on flight)

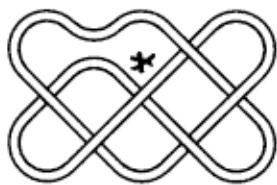
52 $B^{14} \ B^{12} \ B^{10} \ B^8 \left(\begin{matrix} + \\ B^6 \end{matrix} \right) B^4 \ B^2 \ 1 \ A^2 \ A^4 \ A^6 \ A^8 \ A^{10} \ A^{12} \ A^{14}$


 $=$ 
 $=$ $\boxed{B^6}$

Compute the Kauffman polynomial for each of the following knots and links:

53

$$B^{14} \quad B^{12} \quad \boxed{+ \quad B^{10} \quad B^8 \quad B^6 \quad B^4 \quad B^2 \quad 1 \quad - \quad A^2 \quad A^4 \quad + \quad A^6 \quad A^8 \quad - \quad A^{10}} \quad A^{12} \quad A^{14}$$



$= A$



$-B^3$



$(-A^4-B^4) \cdot (-A^7-A^3-B^5)$

$= \boxed{B^{10}+B^2-A^2+A^6-A^{10}}$

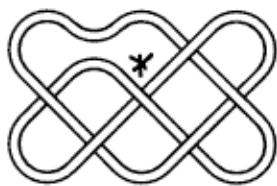


$\left(-A^{10}+A^6+B^2 \right) \left(-A^2+B^2+B^{10} \right) B^2$

(mirror of 48)

54

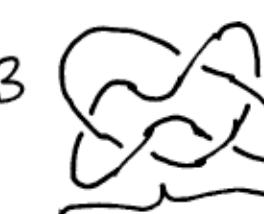
$$B^{14} \quad B^{12} \quad \boxed{+ \quad B^{10} \quad B^8 \quad B^6 \quad B^4 \quad B^2 \quad 1 \quad + \quad A^2 \quad A^4 \quad - \quad A^6 \quad A^8 \quad + \quad A^{10}} \quad A^{12} \quad A^{14}$$



$= A$



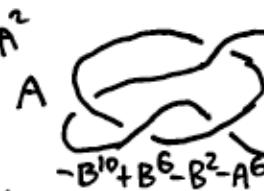
$-B^2 \cdot (-A^4-B^4) \cdot (B^7-B^3-A^5)$



$= \boxed{B^{14}-2B^{10}+2B^6-2B^2+2A^2-A^6+A^{10}}$

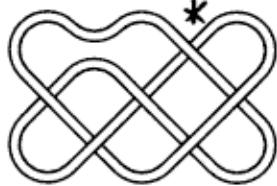
$-B^2+A^2+A^{10}-B^{10}+B^6+A^2$

(mirror of 47)

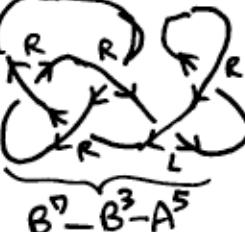


55

$$B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad \boxed{B^6 \quad + \quad B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \quad - \quad A^6 \quad A^8 \quad A^{10} \quad A^{12} \quad A^{14}}$$



$= A$



$B^7-B^3-A^5$



$(-A^3)(-A^4-B^4)$

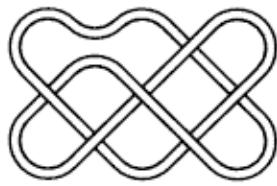
$= \boxed{B^6}$



B^6

56

$$B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad \boxed{B^6 \quad + \quad B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \quad - \quad A^6 \quad A^8 \quad A^{10} \quad A^{12} \quad A^{14}}$$



$=$



$= \boxed{B^6}$

Compute the Kauffman polynomial for each of the following knots and links:

57 $B^{14} \quad B^{12} \quad \boxed{-B^{10} \quad B^8 \quad B^6 \quad B^4 \quad B^2 \quad 1 \quad A^2}$ $A^4 \quad A^6 \quad A^8 \quad A^{10} \quad A^{12} \quad A^{14}$

 $= -B^3(B^7 - B^3 - A^5)$
 $= \boxed{-B^{10} + B^6 + A^2}$

58 $B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad \boxed{+ \quad B^6}$ $B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \quad A^6 \quad A^8 \quad A^{10} \quad A^{12} \quad A^{14}$

 $= \boxed{B^6}$

59 $B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad \boxed{+ \quad B^6}$ $B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \quad A^6 \quad A^8 \quad A^{10} \quad A^{12} \quad A^{14}$

 $= \boxed{B^6}$

60 $B^{14} \quad B^{12} \quad B^{10} \quad B^8 \quad B^6 \quad B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \quad \boxed{+ \quad A^6}$ $A^8 \quad A^{10} \quad A^{12} \quad A^{14}$

 $= \boxed{A^6}$

(If you make it to problems 61-80, check your work by doing each problem two different ways; your answers should agree.)