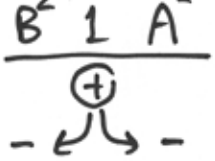
Rules:  $AB = 1$

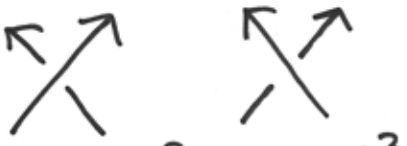
(Americans approaching intersection on overpass veer off to right.
Brits veer off to left. This convention is opposite that in the literature.)


extra loops $\longrightarrow C$

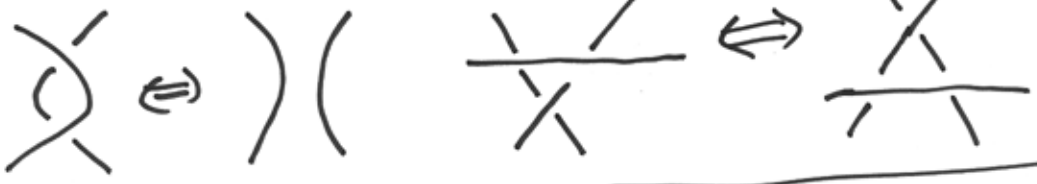
$C = -A^2 - B^2$
represented graphically as 

For trivial knot, Kauffman = Writhe.
To compute writhe, pick orientation,

~~LR~~ cancel
~~AB~~ cancel
$RRL = R = \boxed{-A^3}$


$L = -B^3$ $R = -A^3$

In general, can slide knot around like a belt between two sheets of glass,
II & III Reidemeister moves OK:

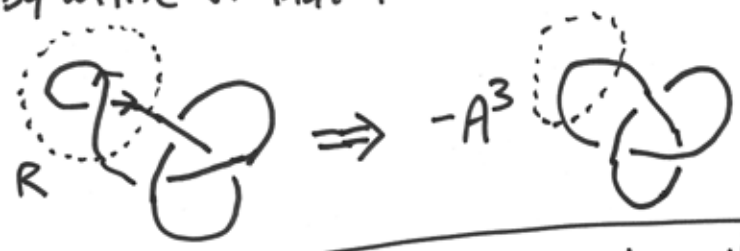


Above example easier this way:

$R = \boxed{-A^3}$

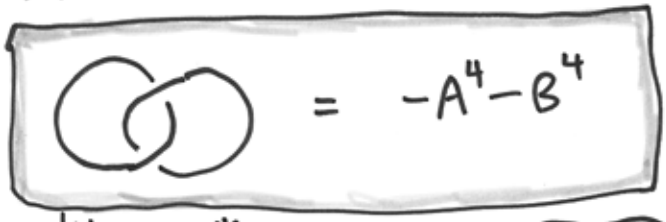
push loop over to simplify knot

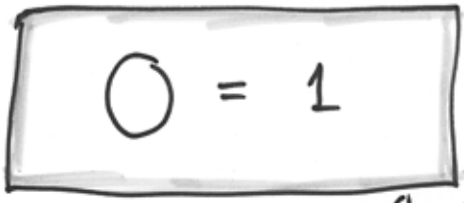
Removing a twist (Reidemeister I) from any knot multiplies Kauffman by writhe of that twist:



Removing a loop from any knot multiplies Kauffman by C:

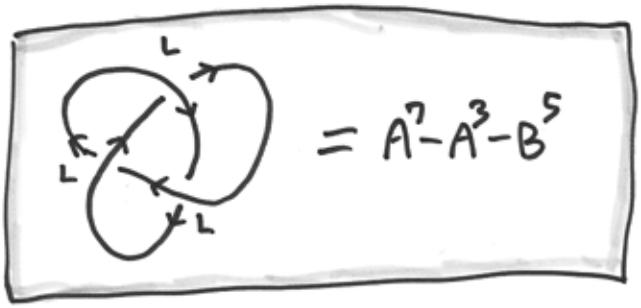
Nice to remember answers when we get down to something familiar:

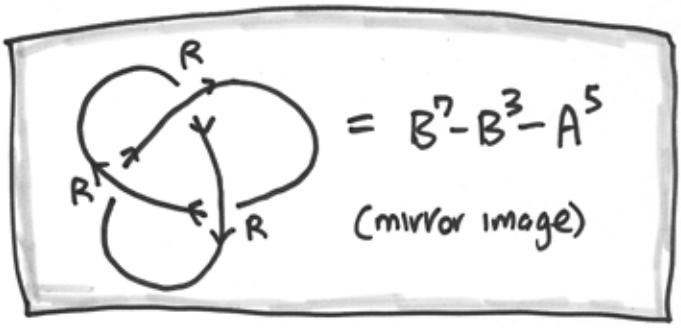




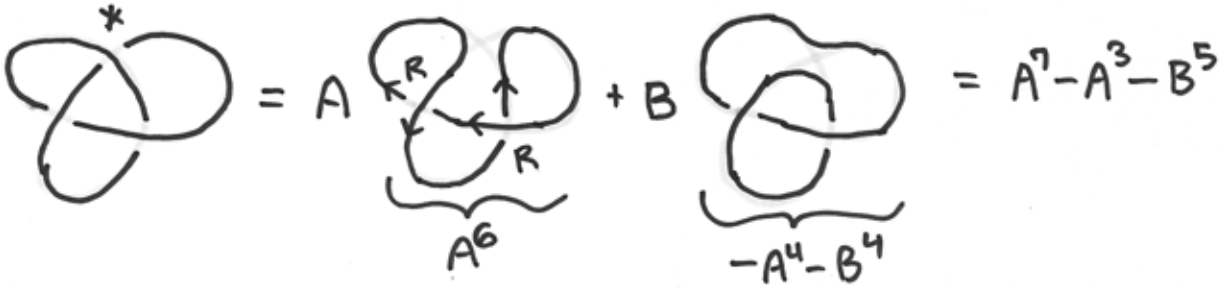
(by convention)
(don't count 1st loop)

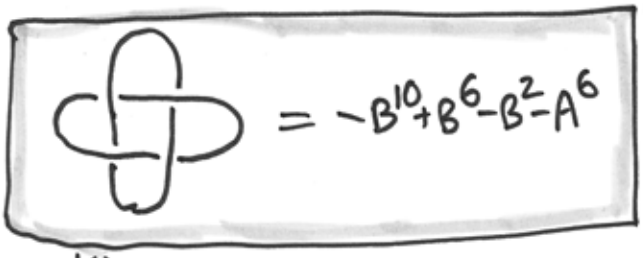
work:

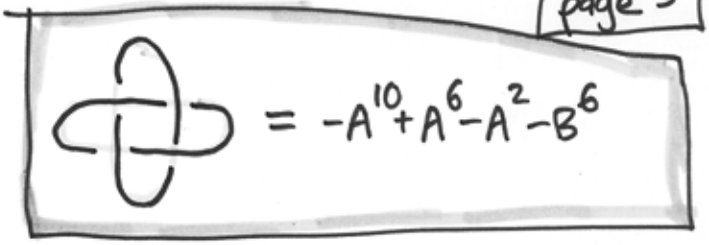




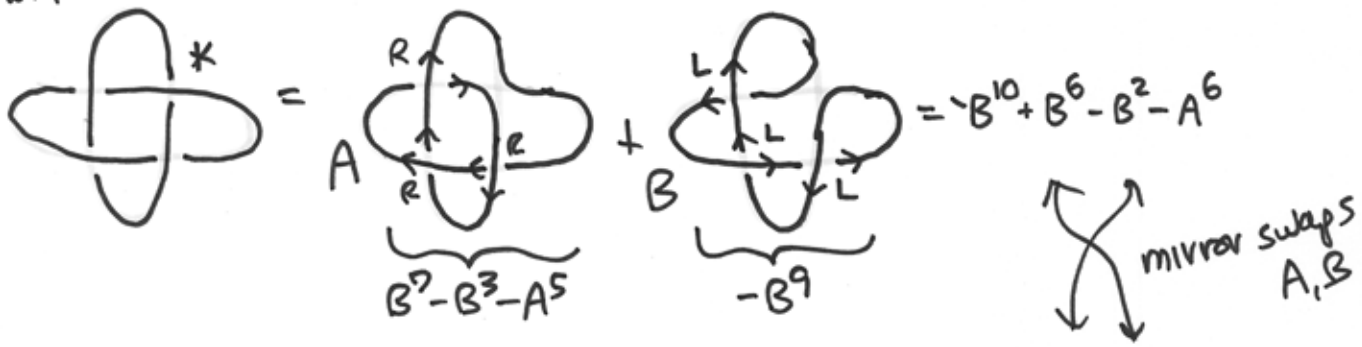
work:

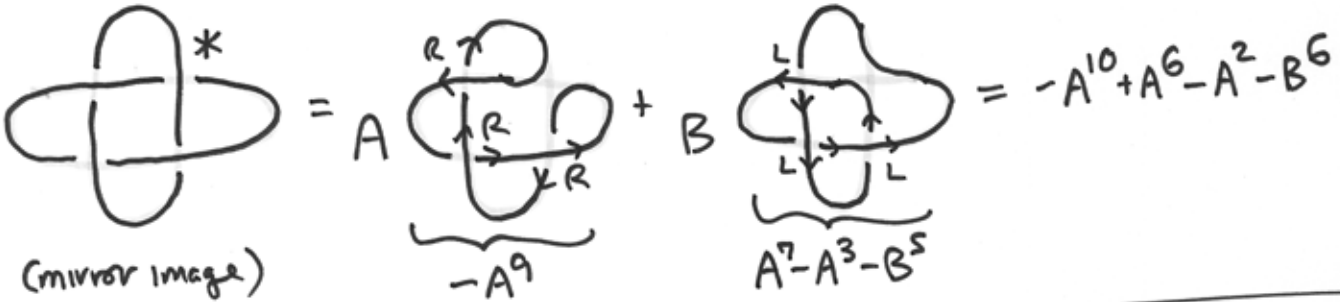




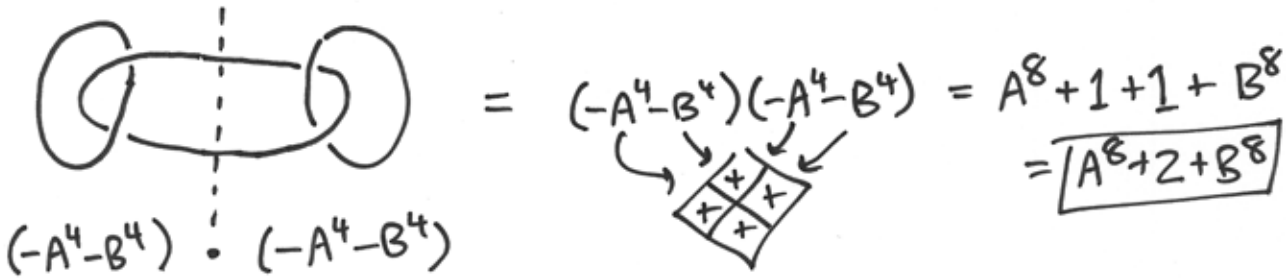


work:





Sum of two knots or links is product of Kauffman polys:



check this example:

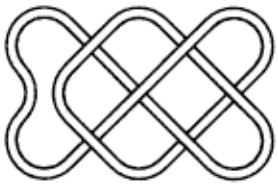
$= A(-A^3)(-A^4-B^4) + B(-B^3)(-A^4-B^4)$

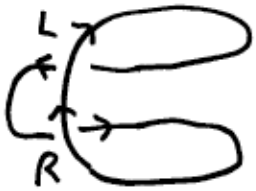
$= A^8 + 1 + 1 + B^8$ ☑

Compute the Kauffman polynomial for each of the following knots and links:

01

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 (1, +) A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}

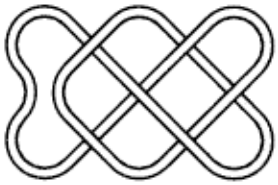




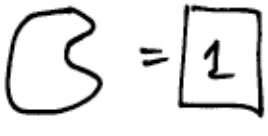
= 1 unknot

02

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 (1, +) A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



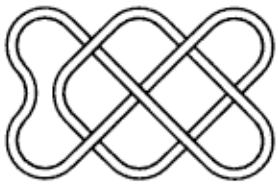
again

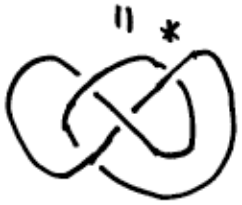


= 1 unknot

03

B^{14} B^{12} B^{10} (B^8 + B^6 − B^4 + B^2 − 1 + A^2 − A^4 + A^6 − A^8 +) A^{10} A^{12} A^{14}



= 

$= A$ 

$+ B$ 

$= B^8 - B^4 + 1 - A^4 + A^8$

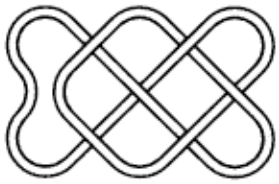
figure-8 knot

$-A^3(-A^4 - B^4)$

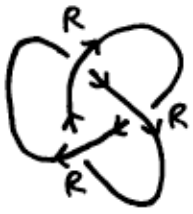
$B^7 - B^3 - A^5$

04

B^{14} B^{12} B^{10} B^8 B^6 (B^4 − B^2 + 1 A^2 − A^4 − A^6 + A^8) A^{10} A^{12} A^{14}



= 

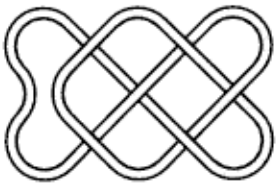
$= -A^3$ 

$= -A^3(B^7 - B^3 - A^5) = -B^4 + 1 + A^8$

Compute the Kauffman polynomial for each of the following knots and links:

05

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}

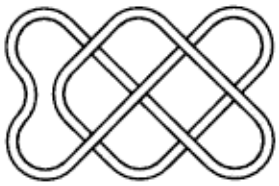


$= -B^3 = -B^3(A^7 - A^3 - B^5)$

$= -A^4 + 1 + B^8$

06

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



$= A + B = B^8 + 1 - A^4$

$-B^3(-B^4 - A^4)$ (trefoil with a twist)

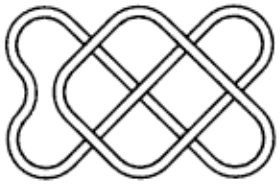
$-B^3$ $A^7 - A^3 - B^5$

// (or) check

$= -A^3$

07

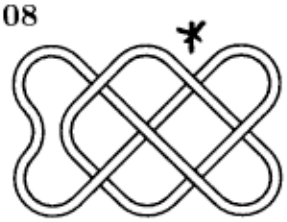
B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



$= = = 1$

08

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



$= A + B = B^8 - B^4 + 2 - A^4 + A^8 - A^{12}$

$+A^8(B^7 - B^3 - A^5)$

2 twists trefoil

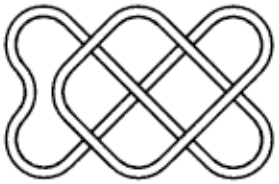
$= -A^3$

$(-B^{10} + B^6 - B^2 - A^6)$

Compute the Kauffman polynomial for each of the following knots and links:

09

$B^{14}\ B^{12}\ B^{10}\ B^8\ B^6\ B^4\ B^2\ \boxed{1}\ A^2\ A^4\ A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$



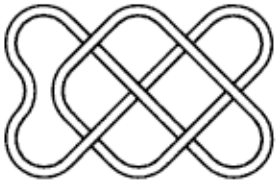
$=$

$=$

$= \boxed{1}$

10

$B^{14}\ B^{12}\ B^{10}\ \boxed{\begin{matrix} + & & - & & + & & - & & + \\ B^8 & B^6 & B^4 & B^2 & 1 & A^2 & A^4 & A^6 & A^8 \end{matrix}}\ A^{10}\ A^{12}\ A^{14}$



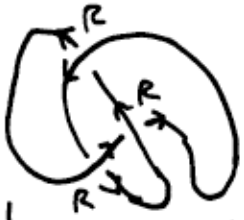
$=$

(looks like figure-8)

$= A$

$-A^3(-A^4-B^4)$

$+ B$

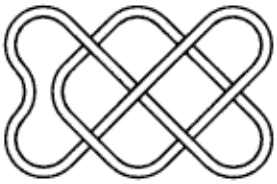


$B^7-B^3-A^5$

$= \boxed{B^8-B^4+1-A^4+A^8}$

11

$B^{14}\ B^{12}\ B^{10}\ \boxed{\begin{matrix} B^8 & B^6 & B^4 & B^2 & 1 & A^2 & A^4 & A^6 & A^8 \\ + & & - & & + & & - & & + \end{matrix}}\ A^{10}\ A^{12}\ A^{14}$



$=$

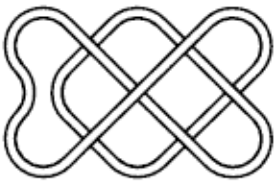
mirror image of 10

$= \boxed{B^8-B^4+1-A^4+A^8}$

figure 8 again

12

$B^{14}\ B^{12}\ B^{10}\ B^8\ B^6\ B^4\ B^2\ \boxed{1}\ A^2\ A^4\ A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$



$=$

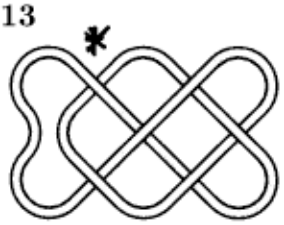
$=$

$= \boxed{1}$

Compute the Kauffman polynomial for each of the following knots and links:

13

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



$= A$  $+ B$  $\Rightarrow$ 

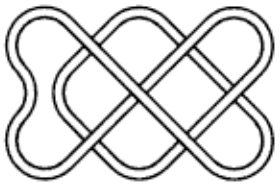
$-A^3(-A^4-B^4)$ (mirror image of 08)

$-B^{12}+B^8-B^4+2-A^4+A^8 = -B^3$  figure-8

$B^8-B^4+1-A^4+A^8$

14

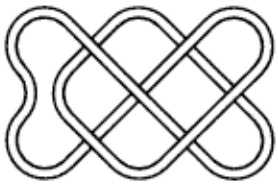
B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



$=$  $=$  $= \boxed{1}$

15

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



$=$  $= A$  $+ B$ 

(trefoil w/a twist)

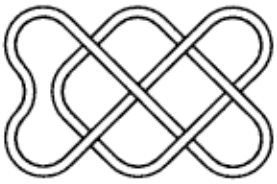
$= \boxed{-B^4+1+A^8}$

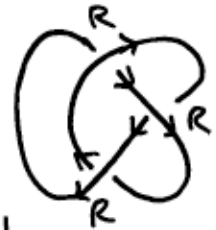
$-A^3(-A^4-B^4)$

$-B^3$

16

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



$=$  $= -A^3$  $= \boxed{-B^4+1+A^8}$

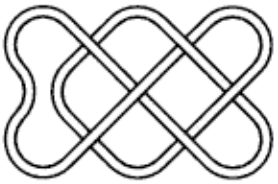
$B^7-B^3-A^5$

again, trefoil w/a twist

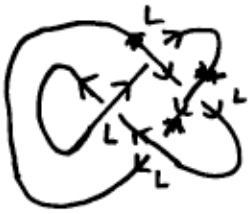
Compute the Kauffman polynomial for each of the following knots and links:

17

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



$=$

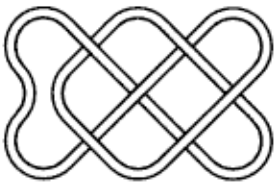


$= -B^3(A^7 - A^3 - B^5) = +B^8 + 1 - A^4$

another twisted trefoil

18

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



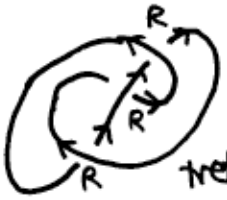
$=$

looks like figure-8

$= A$

$-A^3(-A^4 - B^4)$

$+ B$



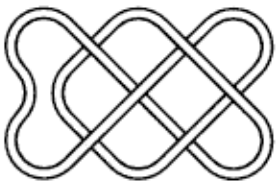
trefoil

$(B^7 - B^3 - A^5)$

$= B^8 - B^4 + 1 - A^4 + A^8$

19

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



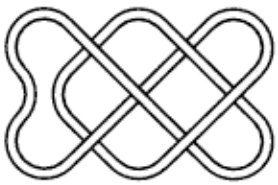
$=$

$=$

$= 1$

20

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



$=$

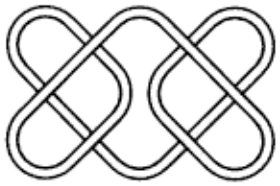
$=$

$= 1$

Compute the Kauffman polynomial for each of the following knots and links:

21

$B^{14}\ B^{12}\ B^{10}\ B^8\ B^6\ B^4\ B^2$ (1, +) $A^2\ A^4\ A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$

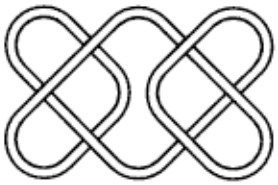


$=$

$= \boxed{1}$

22

$B^{14}\ B^{12}\ B^{10}\ B^8\ B^6\ B^4\ B^2$ (1, +) $A^2\ A^4\ A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$

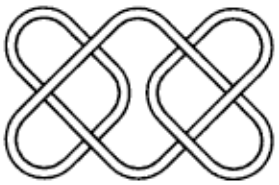


$=$

$= \boxed{1}$

23

$B^{14}\ B^{12}\ B^{10}$ [+ + −; $B^8\ B^6\ B^4\ B^2\ 1\ A^2\ A^4$] $A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$



$=$

$= -B^3$

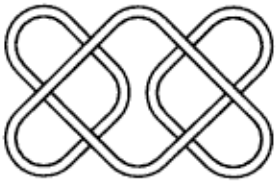
$= \boxed{B^8+1-A^4}$

$A^7-A^3-B^5$

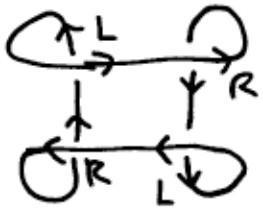
twisted trefoil

24

$B^{14}\ B^{12}\ B^{10}\ B^8\ B^6\ B^4\ B^2$ (1, +) $A^2\ A^4\ A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$



$=$

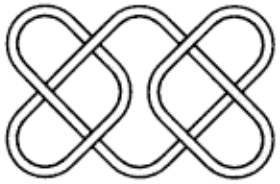


$= \boxed{1}$

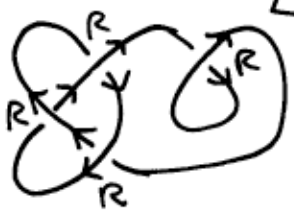
Compute the Kauffman polynomial for each of the following knots and links:

25

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



$=$

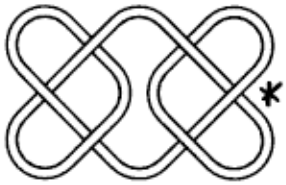


$= -A^3(B^7 - B^3 - A^5)$

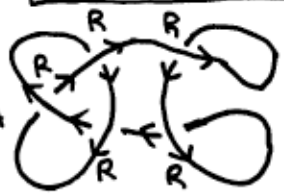
$= \boxed{-B^4 + 1 + A^8}$

26

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



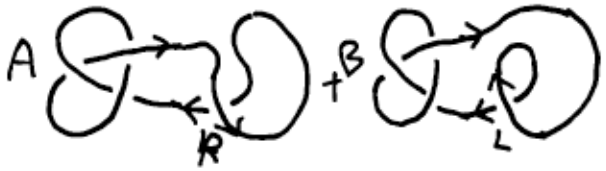
$= A$



$+B$

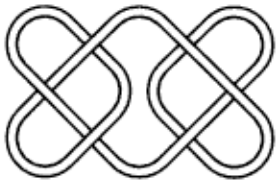
$= \boxed{-B^{12} + B^8 - B^4 + 3 - A^4 + A^8 - A^{12}}$

$= (B^7 - B^3 - A^5)(A^7 - A^3 - B^5)$

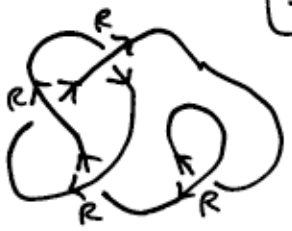


27

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



$=$

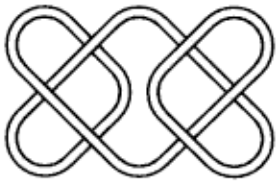


$= -A^3(B^7 - B^3 - A^5)$

$= \boxed{-B^4 + 1 + A^8}$

28

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



$=$

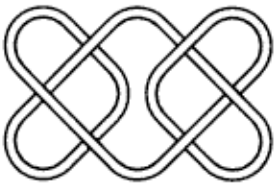
$= \boxed{1}$

Compute the Kauffman polynomial for each of the following knots and links:

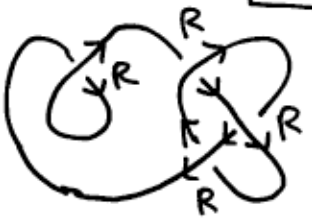
29

B^{14} B^{12} B^{10} B^8 B^6 $\boxed{B^4\ B^2\ 1\ A^2\ A^4\ A^6\ A^8}$ A^{10} A^{12} A^{14}

(− + +)



$=$

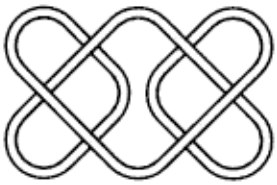


$= -A^3(B^7 - B^3 - A^5)$

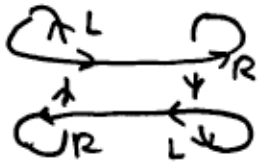
$= \boxed{-B^4 + 1 + A^8}$

30

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 $\underset{+}{1}$ A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



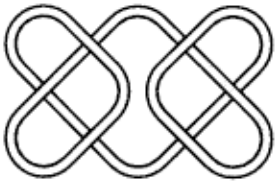
$=$



$= \boxed{1}$

31

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 $\underset{+}{1}$ A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



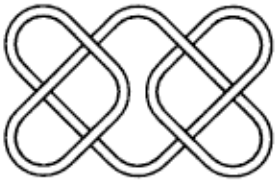
$=$

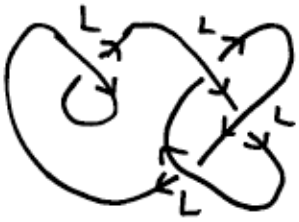
$= \boxed{1}$

32

B^{14} B^{12} B^{10} $\boxed{B^8\ B^6\ B^4\ B^2\ 1\ A^2\ A^4}$ A^6 A^8 A^{10} A^{12} A^{14}

(+ + −)





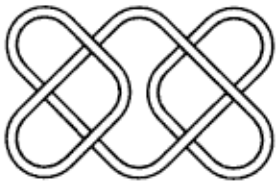
$= -B^3(A^7 - A^3 - B^5)$

$= \boxed{B^8 + 1 - A^4}$

Compute the Kauffman polynomial for each of the following knots and links:

33

$B^{14}\ B^{12}\ B^{10}\ B^8\ B^6\ B^4\ B^2$ (1, +) $A^2\ A^4\ A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$

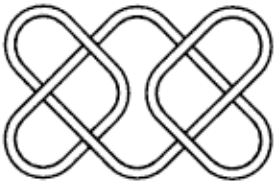


$=$

$= \boxed{1}$

34

$B^{14}\ B^{12}\ B^{10}$ (+ $B^8\ B^6\ B^4\ B^2$ + $1\ A^2$ − A^4) $A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$



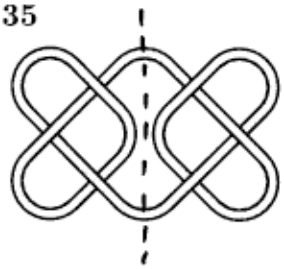
$=$

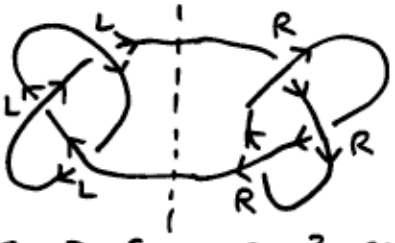
$= -B^3(A^7 - A^3 - B^5)$

$= \boxed{B^8 + 1 - A^4}$

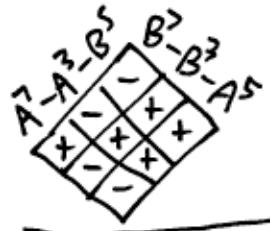
35

B^{14} (− B^{12} + B^{10} − $B^8\ B^6\ B^4\ B^2$ + 1 − A^2 + $A^4\ A^6$ − $A^8\ A^{10}$ + A^{12}) A^{14}





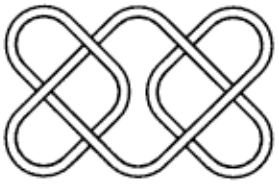
$(A^7 - A^3 - B^5) \cdot (B^7 - B^3 - A^5)$



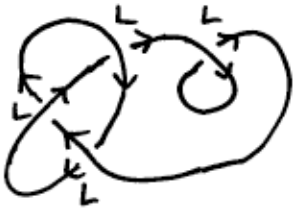
$= \boxed{-B^{12} + B^8 - B^4 + 3 - A^4 + A^8 - A^{12}}$

36

$B^{14}\ B^{12}\ B^{10}$ ($B^8\ B^6\ B^4\ B^2$ + $1\ A^2$ + A^4 −) $A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$



$=$



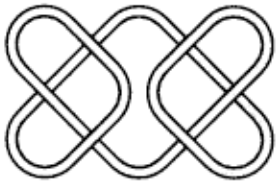
$= -B^3(A^7 - A^3 - B^5)$

$= \boxed{B^8 + 1 - A^4}$

Compute the Kauffman polynomial for each of the following knots and links:

37

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}

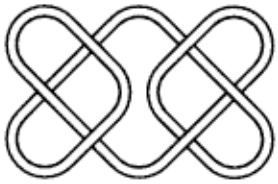


$=$

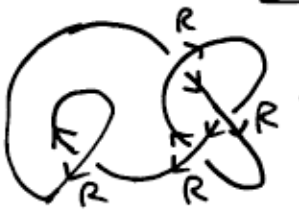
$= \boxed{1}$

38

B^{14} B^{12} B^{10} B^8 B^6 $\boxed{B^4 \quad B^2 \quad 1 \quad A^2 \quad A^4 \quad A^6 \quad A^8}$ A^{10} A^{12} A^{14}



$=$

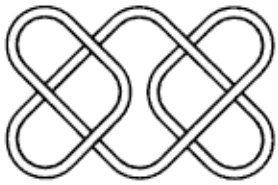


$= -A^3(B^7 - B^3 + A^5)$

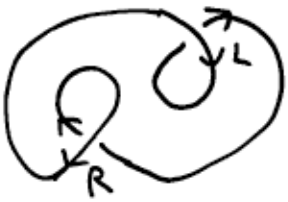
$= \boxed{A^8 + 1 - B^4}$

39

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



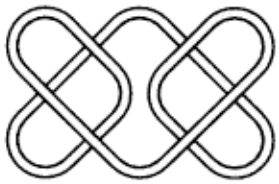
$=$

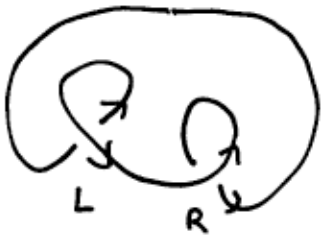


$= \boxed{1}$

40

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



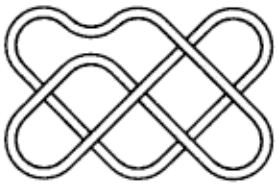


$= \boxed{1}$

Compute the Kauffman polynomial for each of the following knots and links:

41

$B^{14}\ B^{12}\ B^{10}\ B^8\ \underset{+}{B^6}\ B^4\ B^2\ 1\ A^2\ A^4\ A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$

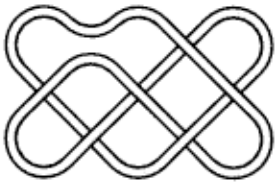


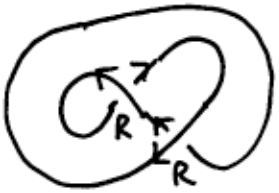
$=$ 

$= \boxed{B^6}$

42

$B^{14}\ B^{12}\ B^{10}\ B^8\ B^6\ B^4\ B^2\ 1\ A^2\ A^4\ \underset{+}{A^6}\ A^8\ A^{10}\ A^{12}\ A^{14}$

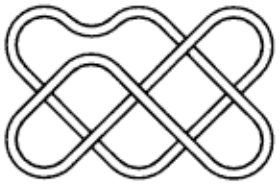


$=$ 

$= \boxed{A^6}$

43

$B^{14}\ B^{12}\ B^{10}\ B^8\ B^6\ B^4\ B^2\ 1\ A^2\ A^4\ \underset{+}{A^6}\ A^8\ A^{10}\ A^{12}\ A^{14}$

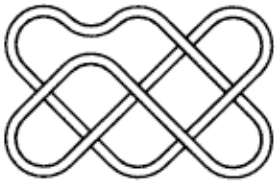


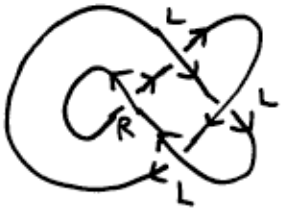
$=$ 

$= \boxed{A^6}$

44

$B^{14}\ B^{12}\ B^{10}\ B^8\ B^6\ B^4\ \underset{+}{B^2}\ 1\ A^2\ A^4\ \underset{+}{A^6}\ A^8\ \underset{-}{A^{10}}\ A^{12}\ A^{14}$



$=$ 

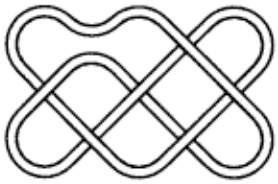
$= -A^3(A^7 - A^3 - B^5)$

$= \boxed{-A^{10} + A^6 + B^2}$

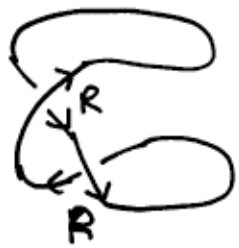
Compute the Kauffman polynomial for each of the following knots and links:

45

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 $\boxed{A^6\ +}$ A^8 A^{10} A^{12} A^{14}



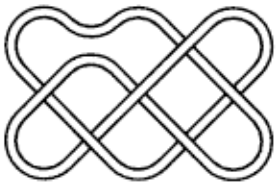
$=$



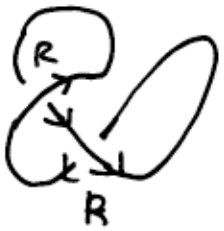
$= \boxed{A^6}$

46

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 $\boxed{A^6\ +}$ A^8 A^{10} A^{12} A^{14}



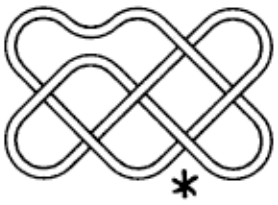
$=$



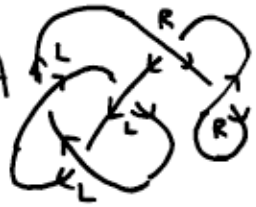
$= \boxed{A^6}$

47

B^{14} B^{12} $\boxed{+}$ B^{10} $\boxed{-}$ B^8 $\boxed{+}$ B^6 $\boxed{=}$ B^4 $\boxed{+}$ B^2 $\boxed{=}$ 1 $\boxed{+}$ A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



$= A$



$+B$

$A^6(A^7-A^3-B^5)$

$B^{10}-B^6+2B^2-2A^2+2A^6-2A^{10}+A^{14}$

A

$+B$

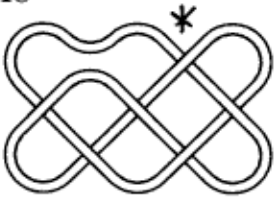
figure-8

$(-A^3)(A^7-A^3+B^5)$

$B^8-B^4+1-A^4+A^8$

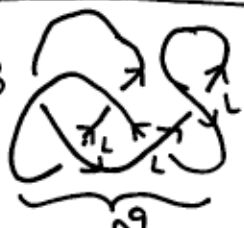
48

B^{14} B^{12} $\boxed{-}$ B^{10} B^8 $\boxed{+}$ B^6 B^4 $\boxed{-}$ B^2 1 $\boxed{+}$ A^2 A^4 A^6 A^8 $\boxed{+}$ A^{10} A^{12} A^{14}



$= A$

$+B$



$= A^2(A^8+2+B^8) - A^2 - B^2 - B^{10}$

$-B^9$

$= \boxed{A^{10}+A^2-B^2+B^6-B^{10}}$

A

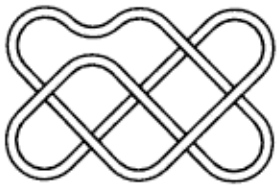
$+B$

$(-A^4-B^4)\cdot(-A^4-B^4)$ $\quad -A^2-B^2$

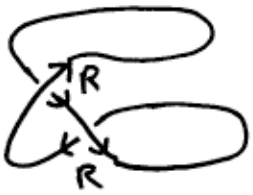
Compute the Kauffman polynomial for each of the following knots and links:

49

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 $\overset{+}{A^6}$ A^8 A^{10} A^{12} A^{14}



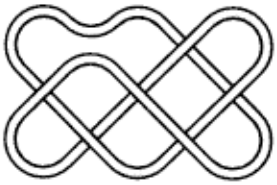
$=$



$= \boxed{A^6}$

50

B^{14} B^{12} B^{10} B^8 B^6 B^4 B^2 1 A^2 A^4 $\overset{+}{A^6}$ A^8 A^{10} A^{12} A^{14}

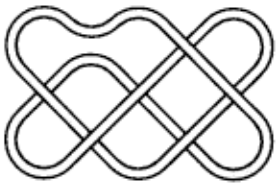


$=$

$= \boxed{A^6}$

51

B^{14} B^{12} B^{10} B^8 $\overset{+}{B^6}$ B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}



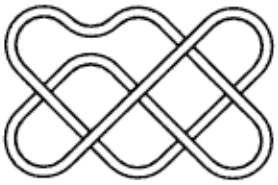
$=$

$= \boxed{B^6}$

(turbulence on Flight)

52

B^{14} B^{12} B^{10} B^8 $\overset{+}{B^6}$ B^4 B^2 1 A^2 A^4 A^6 A^8 A^{10} A^{12} A^{14}

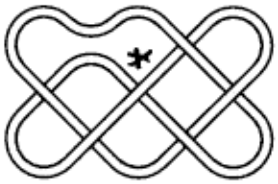


$=$

$= \boxed{B^6}$

Compute the Kauffman polynomial for each of the following knots and links:

53 $B^{14}\ B^{12}\ B^{10}\ B^8\ B^6\ B^4\ B^2\ 1\ A^2\ A^4\ A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$



$= A$  $+ B$ 

$-B^3$

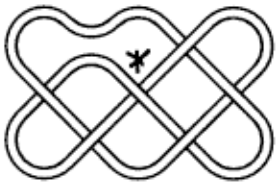
$(-A^4-B^4)\cdot(A^7-A^3-B^5)$

$(-A^{10}+A^6+B^2)$
$-A^2+B^2+B^{10}$ B^2

$= B^{10}+B^2-A^2+A^6-A^{10}$

(mirror of 48)

54 $B^{14}\ B^{12}\ B^{10}\ B^8\ B^6\ B^4\ B^2\ 1\ A^2\ A^4\ A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$



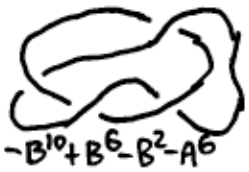
$= A$  $+ B$ 

$= B^{14}-2B^{10}+2B^6-2B^2+2A^2-A^6+A^{10}$

$(-A^4-B^4)\cdot(B^7-B^3-A^5)$

$-B^7+A^2+A^{10}-B^{10}+B^6+A^2$

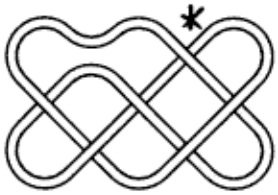
(mirror of 47)

A 

$-B^{10}+B^6-B^2-A^6$

$+B$  B^{12}

55 $B^{14}\ B^{12}\ B^{10}\ B^8\ B^6\ B^4\ B^2\ 1\ A^2\ A^4\ A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$



$= A$  $+ B$ 

$= B^6$

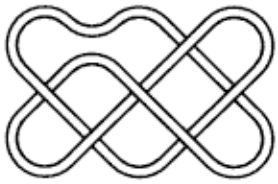
$B^7-B^3-A^5$

$(-A^3)(-A^4-B^4)$

‖ Oh...

 B^6

56 $B^{14}\ B^{12}\ B^{10}\ B^8\ B^6\ B^4\ B^2\ 1\ A^2\ A^4\ A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$



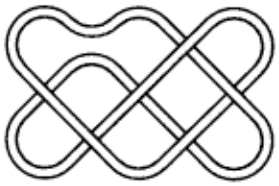
$=$ 

$= B^6$

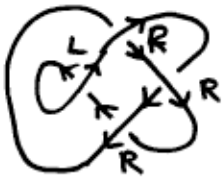
Compute the Kauffman polynomial for each of the following knots and links:

57

$B^{14}\ B^{12}\ \boxed{B^{10}\ B^8\ B^6\ B^4\ B^2\ 1\ A^2}\ A^4\ A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$



$=$

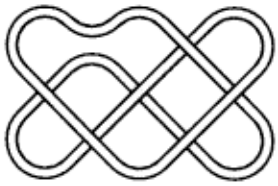


$= -B^3(B^7 - B^3 - A^5)$

$= \boxed{-B^{10} + B^6 + A^2}$

58

$B^{14}\ B^{12}\ B^{10}\ B^8\ \boxed{B^6}\ B^4\ B^2\ 1\ A^2\ A^4\ A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$

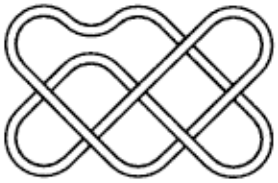


$=$

$= \boxed{B^6}$

59

$B^{14}\ B^{12}\ B^{10}\ B^8\ \boxed{B^6}\ B^4\ B^2\ 1\ A^2\ A^4\ A^6\ A^8\ A^{10}\ A^{12}\ A^{14}$

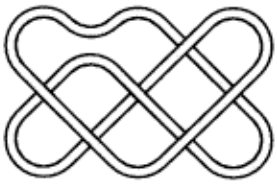


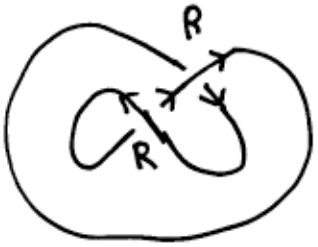
$=$

$= \boxed{B^6}$

60

$B^{14}\ B^{12}\ B^{10}\ B^8\ B^6\ B^4\ B^2\ 1\ A^2\ A^4\ \boxed{A^6}\ A^8\ A^{10}\ A^{12}\ A^{14}$





$= \boxed{A^6}$

(If you make it to problems 61-80, check your work by doing each problem two different ways; your answers should agree.)