Second Midterm

Linear Algebra, Dave Bayer, March 30, 2004

Name: ____________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Let A be the matrix

$$A \;=\; \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 2 & 0 \\ -1 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & -1 & 1 & -2 & 1 \end{bmatrix}.$$

Compute the row space and column space of A.

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[2] Let

$$\mathbf{v}_1 = (1,1,0,0),\ \mathbf{v}_2 = (1,0,1,0),\ \mathbf{v}_3 = (1,0,0,-1),\ \mathbf{v}_4 = (0,1,-1,0),\ \mathbf{v}_5 = (0,1,0,1).$$

Find a basis for the subspace $V \subset \mathbb{R}^4$ spanned by $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$, and $\mathbf{v}_5$. Extend this basis to a basis for $\mathbb{R}^4$.

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[3] Let V be the vector space of all polynomials $f(x)$ of degree ≤ 3. Find a basis for the subspace W defined by

$$f(-1) = f(0), \quad f(-1) = f(1), \quad f(0) = f(1).$$

Extend this basis to a basis for V.

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[4] Let $\mathbf{v}_1 = (1, -1)$ and $\mathbf{v}_2 = (1, 1)$. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that

$$L(\mathbf{v}_1) = 3\mathbf{v}_1 - \mathbf{v}_2, \quad L(\mathbf{v}_2) = 3\mathbf{v}_2 - \mathbf{v}_1.$$

Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1, 0)$, $\mathbf{e}_2 = (0, 1)$.

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[5] Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $L(\mathbf{v}) = -\mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $V \subset \mathbb{R}^3$ defined by $x + y + z = 0$, and $L(\mathbf{v}) = \mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $W \subset \mathbb{R}^3$ defined by $x = y = 0$. Find a matrix that represents L with respect to the usual basis

$$\mathbf{e}_1 = (1, 0, 0), \quad \mathbf{e}_2 = (0, 1, 0), \quad \mathbf{e}_3 = (0, 0, 1).$$

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Problem: ______

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