## Exam 2

Linear Algebra, Dave Bayer, March 29, 2001

Name:
ID: $\qquad$ School: $\qquad$

| $[\mathbf{1}](6 \mathrm{pts})$ | $[\mathbf{2}](6 \mathrm{pts})$ | $[\mathbf{3}](6 \mathrm{pts})$ | $[\mathbf{4}](6 \mathrm{pts})$ | $[\mathbf{5}](6 \mathrm{pts})$ | TOTAL |
| :--- | :--- | :--- | :--- | :--- | :--- |
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|  |  |  |  |  |  |

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.
[1] Let $A$ be the matrix

$$
A=\left[\begin{array}{rrrr}
1 & 2 & 3 & 4 \\
2 & 4 & 6 & 8 \\
3 & 6 & 9 & 12
\end{array}\right]
$$

Compute the row space and column space of $A$.
$\qquad$
[2] Let

$$
\mathbf{v}_{1}=(1,1,0,-1), \quad \mathbf{v}_{2}=(1,0,1,-1), \quad \mathbf{v}_{3}=(0,1,1,-1), \quad \mathbf{v}_{4}=(1,-1,0,0) .
$$

Find a basis for the subspace $V \subset \mathbb{R}^{4}$ spanned by $\mathbf{v}_{1}, \mathbf{v}_{2}, \mathbf{v}_{3}$, and $\mathbf{v}_{4}$.
[3] Let $V$ be the vector space of all polynomials $f(x)$ of degree $\leq 3$. Let $W \subset V$ be the set of all polynomials $f$ in $V$ which satisfy $f^{\prime}(1)=1$. Is $W$ is a subspace of $V$ ? Why or why not?
[4] Let $\mathbf{v}_{1}=(1,2)$ and $\mathbf{v}_{2}=(1,3)$. Let $L: \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}$ be the linear transformation such that

$$
L\left(\mathbf{v}_{1}\right)=\mathbf{v}_{1}, \quad L\left(\mathbf{v}_{2}\right)=\mathbf{v}_{1}+\mathbf{v}_{2} .
$$

Find a matrix that represents $L$ with respect to the usual basis $\mathbf{e}_{1}=(1,0), \mathbf{e}_{2}=(0,1)$.
[5] Let $L: \mathbb{R}^{3} \rightarrow \mathbb{R}^{3}$ be the linear transformation such that $L(\mathbf{v})=\mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $V \subset \mathbb{R}^{3}$ defined by $x+y=z$, and $L(\mathbf{v})=\mathbf{0}$ for all $\mathbf{v}$ belonging to the subspace $W \subset \mathbb{R}^{3}$ defined by $x=y=z$. Find a matrix that represents $L$ with respect to the usual basis

$$
\mathbf{e}_{1}=(1,0,0), \quad \mathbf{e}_{2}=(0,1,0), \quad \mathbf{e}_{3}=(0,0,1)
$$

Problem:

