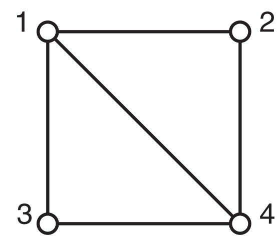
Practice Exam 1

[**1**] Solve the following system of equations:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 6 \end{bmatrix}$$

[**2**] Compute a matrix giving the number of walks of length 4 between pairs of vertices of the following graph:



[**3**] Express the following matrix as a product of elementary matrices:

$$\begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 \end{bmatrix}$$

[**4**] Compute the determinant of the following 4×4 matrix:

$$\begin{bmatrix} \lambda & 1 & 0 & 0 \\ 1 & \lambda & 1 & 0 \\ 0 & 1 & \lambda & 1 \\ 0 & 0 & 1 & \lambda \end{bmatrix}$$

What can you say about the determinant of the $n \times n$ matrix with the same pattern?

[**5**] Use Cramer's rule to give a formula for w in the solution to the following system of equations:

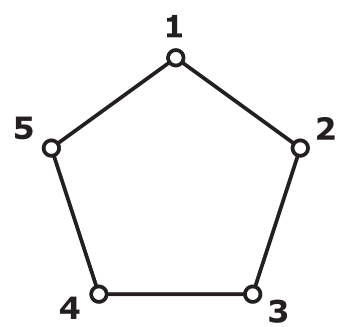
$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Exam 1

[1] Solve the following system of equations:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

[2] Compute matrices giving the number of walks of lengths 1, 2, and 3 between pairs of vertices of the following graph:



[3] Express the following matrix as a product of elementary matrices:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

[4] Compute the determinant of the following 4×4 matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 2 & 0 & 2 \\ 3 & 0 & 3 & 3 \\ 0 & 4 & 4 & 4 \end{bmatrix}$$

What can you say about the determinant of the $n \times n$ matrix with the same pattern?

[5] Use Cramer's rule to give a formula for the solution to the following system of equations:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2a \\ 2b \\ 2c \end{bmatrix}$$