Final Exam

Linear Algebra, Dave Bayer, May 8, 2001

Name: ______________________________

ID: ____________________ **School:** ____________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	[**6**] (6 pts)	[**7**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Let V be the vector space of all polynomials $f(x)$ of degree ≤ 3. Let $W \subset V$ be the subpace of all *odd* polynomials f in V, i.e. those polynomials $f(x)$ in V which satisfy $f(-x) = -f(x)$. Find a basis for W. Extend this basis to a basis for V.

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answer:

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work:

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[**2**] Find the determinants of the matrices

$$A_3 = \begin{bmatrix} \lambda & 0 & -1 \\ 0 & \lambda{-}1 & 0 \\ -1 & 0 & \lambda \end{bmatrix}, \quad A_4 = \begin{bmatrix} \lambda & 0 & 0 & -1 \\ 0 & \lambda & -1 & 0 \\ 0 & -1 & \lambda & 0 \\ -1 & 0 & 0 & \lambda \end{bmatrix}.$$

What is the determinant of the $n \times n$ matrix A_n with the same pattern?

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answer:

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work:

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[**3**] Find an orthogonal basis for the subspace $w - x + y - z = 0$ of $\mathbb{R}^4$.

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answer:

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work:

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[**4**] By least squares, find the equation of the form $y = ax + b$ which best fits the data $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (1, 2)$, $(x_3, y_3) = (2, 1)$.

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answer:

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work:

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[**5**] Find (s, t) so $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

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answer:

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work:

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[**6**] Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Find the eigenvalues and eigenvectors of A.

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answer:

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work:

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[7] Let $A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$. Find the matrix exponential e^{At}.

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answer:

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work:

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