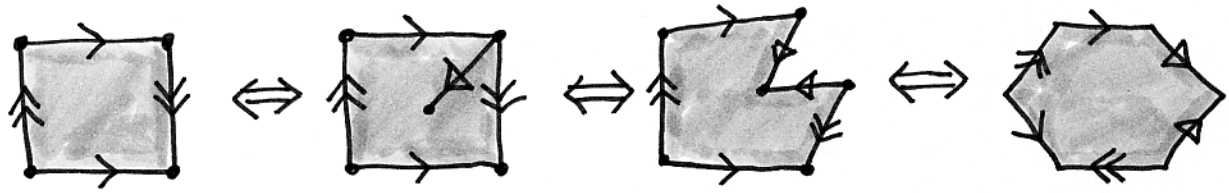
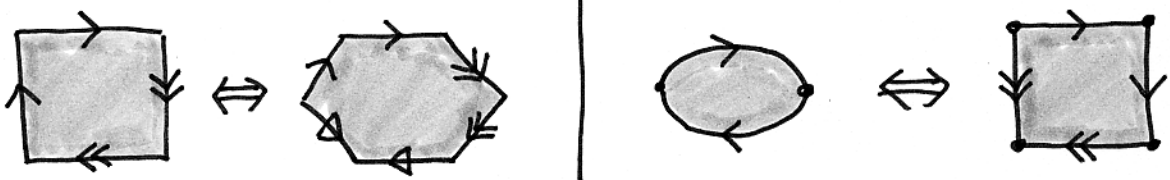
When do two gluing diagrams yield the same surface?

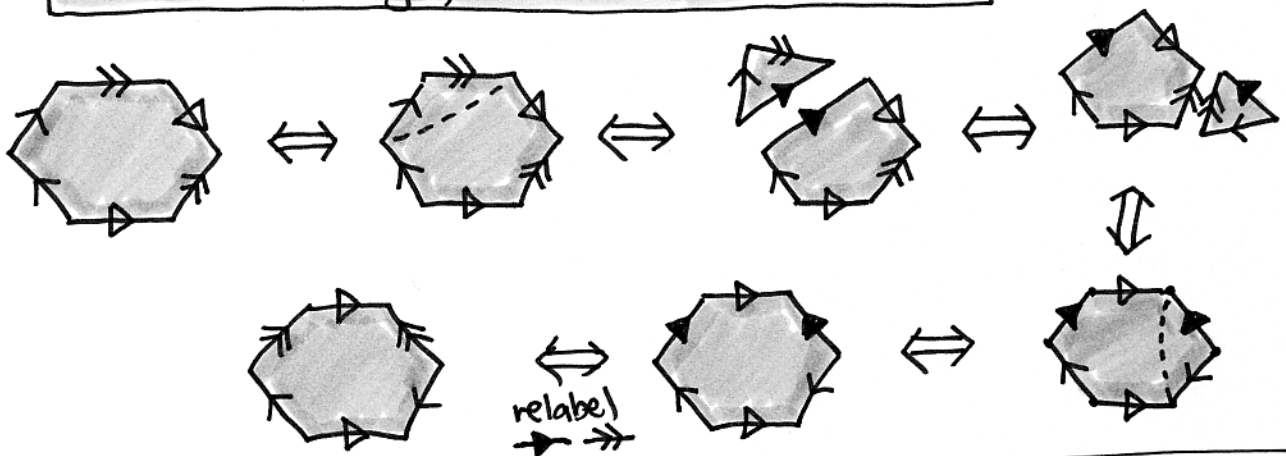
There are two basic "moves" which change a gluing diagram into a similar diagram yielding the same surface. Any two diagrams that yield the same surface, can be changed into one another by a sequence of such moves.

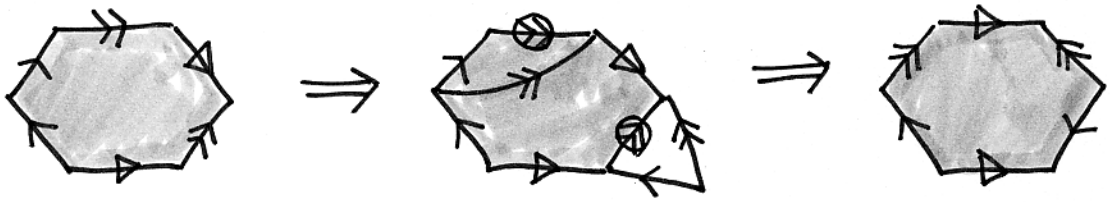
① **Tear a new pair of edges** (or "untear" an existing pair of edges)





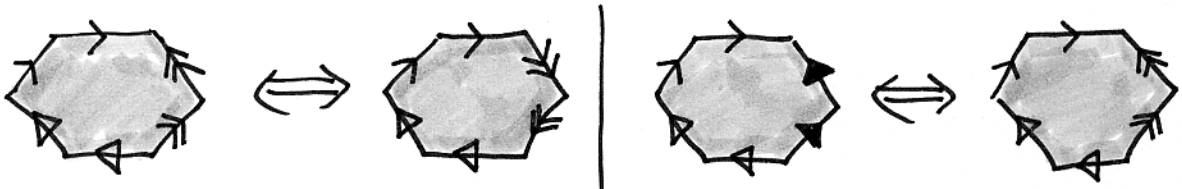
② **Tear off a triangle, and reattach it elsewhere**



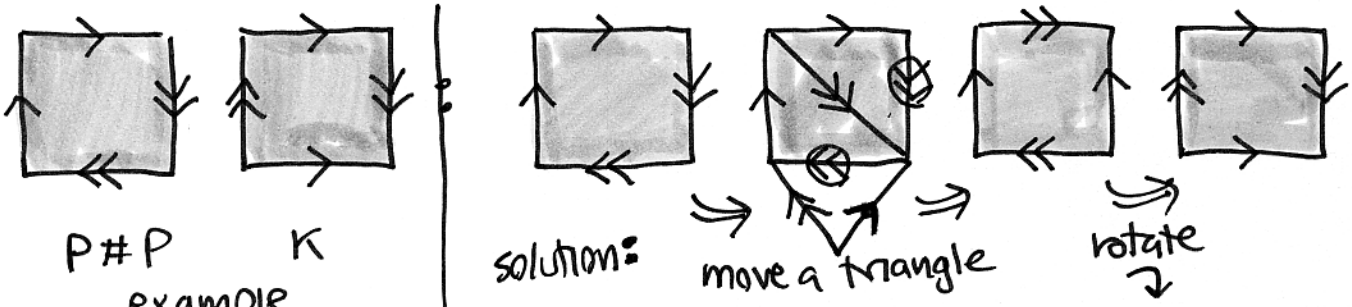


To figure this out with one intermediate drawing, draw the tear, add the triangle to a matching edge, copy the label to the cut, imagine moving the triangle and copy the labels. Redraw.
(We circled the old → as ⊗ to tell it apart.)

③ One can also reverse a pair of edges, or change their symbol.

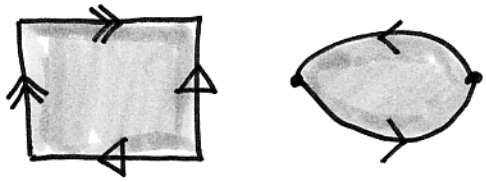


Show that the following two surfaces are the same:

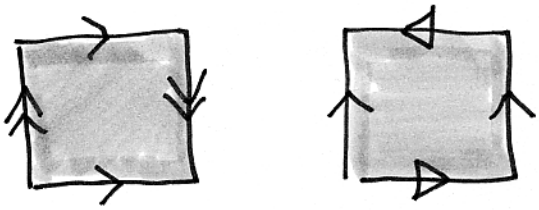


Exercises:

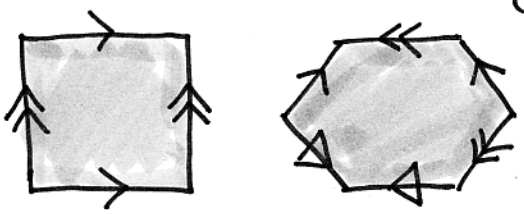
(a)



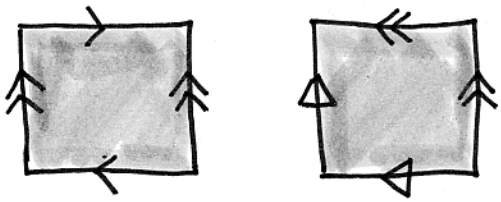
(b)



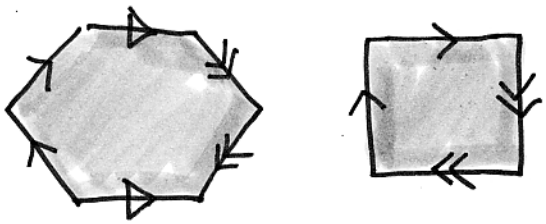
(c)



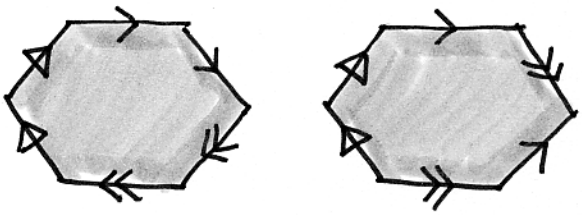
(d)



(e)



(f)



If you can do all these, make up several problems not on this list, that you're expecting might be on the test.