Exam 1

Calculus IIIA, Dave Bayer, February 15, 2001

Name: ANSWERS

ID: ______ **School:** ______

[1] (6 pts)	[2] (6 pts)	[3] (6 pts)	[4] (6 pts)	[5] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[1] Find the area under the parametrized curve

$$x = e^t, \quad y = e^{2t}, \quad 0 \le t \le \ln(2).$$

$$\int_0^{\ln 2} y\,dx = \int_0^{\ln 2} e^{2t}(e^t\,dt) = \int_0^{\ln 2} e^{3t}\,dt$$

$$= \frac{1}{3}e^{3t}\Big|_0^{\ln 2} = \frac{1}{3}\left(e^{3\ln 2} - 1\right)$$

$$= \frac{1}{3}(8-1) = \boxed{\frac{7}{3}}$$

or, $\cancel{u = e^{2t}}$ $\cancel{du = 2e}$ $\boxed{\begin{array}{l} u = e^t \\ du = e^t\,dt \end{array}}$

(notice $y = x^2$!)

$$\begin{array}{l} t = \ln 2 \\ t = 0 \end{array} \Rightarrow \begin{array}{l} u = 2 \\ u = 1 \end{array}$$

$$\int_{t=0}^{t=\ln 2} (e^t)^2 e^t\,dt = \underbrace{\int_{u=1}^{u=2}}_{\text{change limits!}} u^2\,du = \frac{1}{3}u^3\Big|_1^2 = \frac{1}{3}[8-1] = \boxed{\frac{7}{3}}$$

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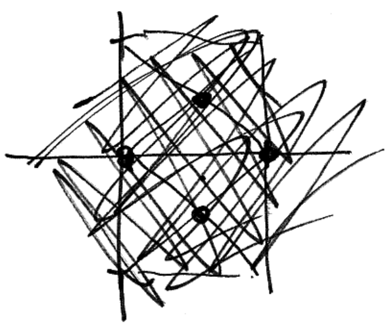
[2] Find the area of one petal of the polar curve

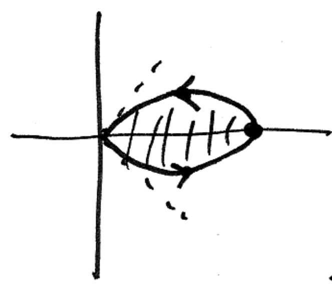
$$r = \cos(2\theta).$$

one petal = r takes one trip away from r=0

e.g. $-\pi/4 \le \theta \le \pi/4$

θ	$-\pi/4$	$-\pi/8$	0	$\pi/8$	$\pi/4$
$r=\cos(2\theta)$	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0





$$A = \frac{1}{2}\int_{-\pi/4}^{\pi/4} r^2\,d\theta$$

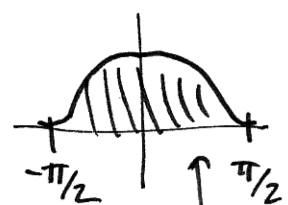
~~$d\theta$ tan~~

$$A = \frac{1}{2}\int_{\theta=-\pi/4}^{\theta=\pi/4} \cos^2(2\theta)\,d\theta = \frac{1}{2}\cdot\frac{1}{2}\int_{u=-\pi/2}^{u=\pi/2} \cos^2 u\,du = \frac{1}{4}\int_{-\pi/2}^{\pi/2} \frac{1}{2}\,du = \boxed{\frac{\pi}{8}}$$

$u = 2\theta$

$2u = 2d\theta$

θ	u
$-\pi/4$	$-\pi/2$
$\pi/4$	$\pi/2$



full piece of $\cos^2 u$

($\cos^2 u + \sin^2 u = 1$, so $\cos^2 u$ acts like $\frac{1}{2}$ on a full piece)

(or do the trig substitution)

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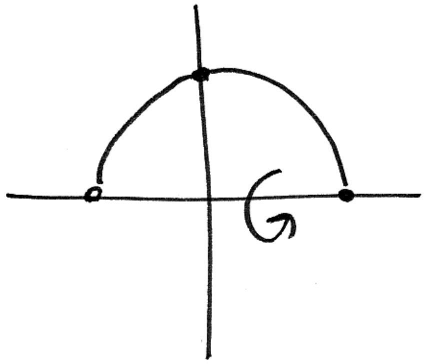
[3] Find the surface area generating by rotating around the x-axis the parametrized curve

$$x = \cos(t), \quad y = \sin(t), \quad 0 \leq t \leq \pi.$$



t	x	y
0	1	0
$\pi/2$	0	1
π	-1	0

$$A = 2\pi \int_a^b y \, ds \qquad ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

$$= \sqrt{\sin^2 t + \cos^2 t} \, dt$$

$$= dt$$

$$A = 2\pi \int_0^\pi \sin(t) dt = 2\pi (-\cos t)\Big|_0^\pi = 2\pi(1+1) = \boxed{4\pi}$$

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[4] Find the arc length of the parametrized curve

$$x = t\cos(t), \quad y = t\sin(t), \quad 0 \le t \le 2\pi.$$

Simplify the integral as far as possible, but do not solve it. Instead, guess its value.

$$L = \int_{t=0}^{t=2\pi} ds = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$\frac{dx}{dt} = \cos(t) - t\sin(t)$$

$$\frac{dy}{dt} = \sin(t) + t\cos(t)$$

$$ds = \sqrt{(\cos(t) - t\sin(t))^2 + (\sin(t) + t\cos(t))^2}\, dt$$

$$= \sqrt{\cos^2 t - 2t\cos t\sin t + t^2\sin^2 t + \sin^2 t + 2t\cos t\sin t + t^2\cos^2 t}\ dt$$

$$= \sqrt{1 + t^2}\ dt$$

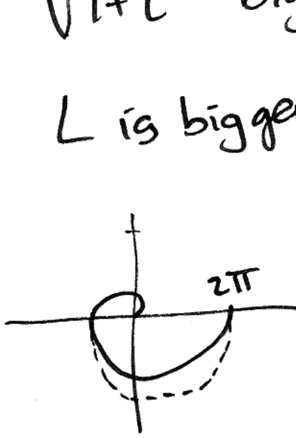
$\boxed{\text{guess } 20}$

$$\Rightarrow L = \int_0^{2\pi} \sqrt{1+t^2}\, dt$$

$$\tfrac{1}{2}t^2\Big|_0^{2\pi} = 2\pi^2$$

$\sqrt{1+t^2}$ bigger than $\sqrt{t^2} = t$

so L is bigger than $\int_0^{2\pi} t\, dt =$



3π

$$\pi\left(\frac{3\pi}{2}\right) = \frac{3\pi^2}{2} \approx 14$$

(correct answer by computer algebra = 21.25)

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[5] Find the arc length of the polar curve

$$r = \theta, \quad 0 \leq \theta \leq 2\pi.$$

Simplify the integral as far as possible, but do not solve it. Instead, guess its value.

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{\theta^2 + 1}\, d\theta$$

same integral as before.

again guess $\approx \boxed{20}$

(correct answer ≈ 21.25 as before)

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